Toward Accurate Coastal Ocean Modeling

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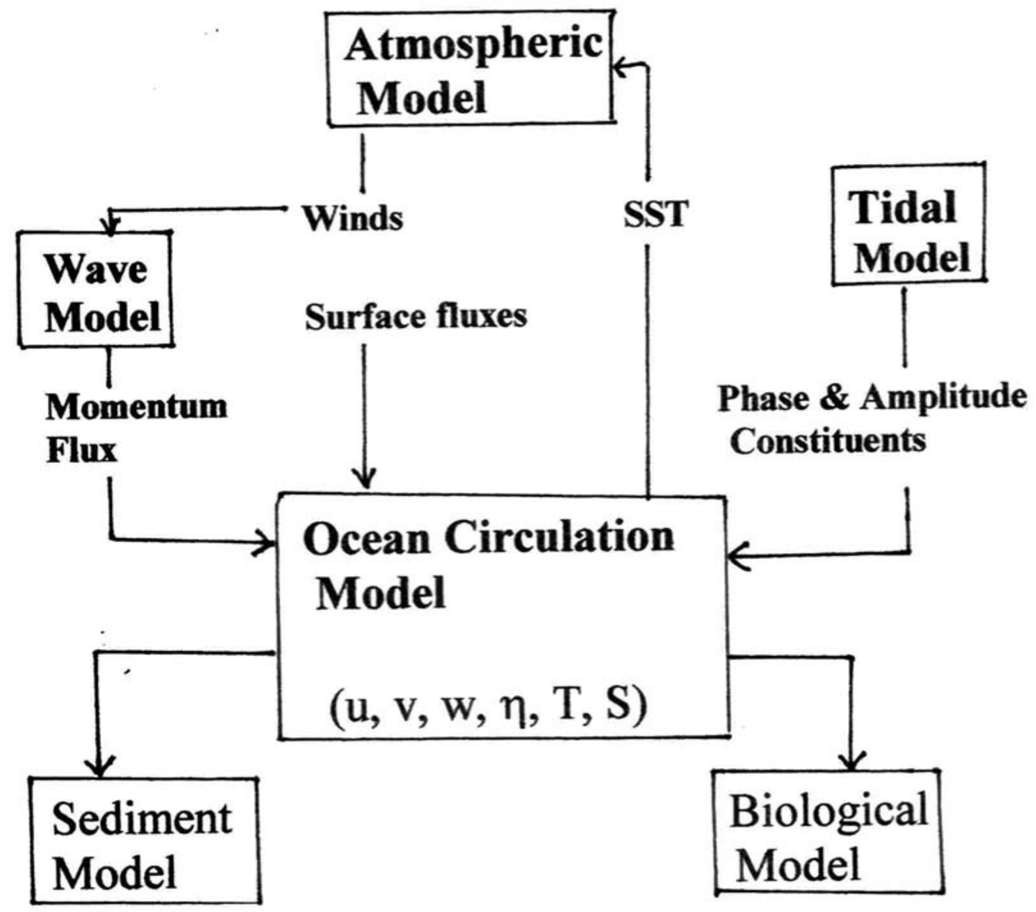
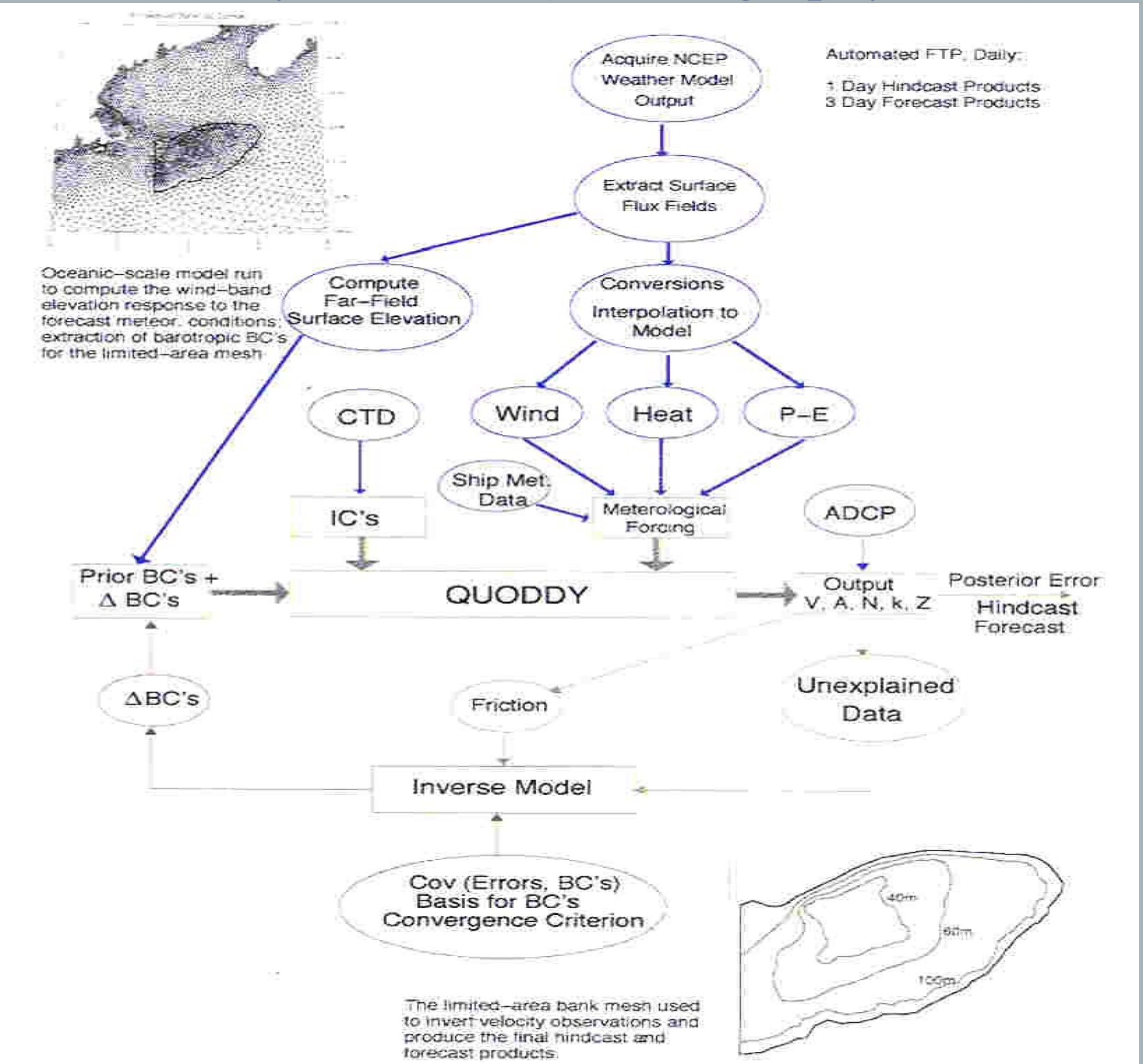
Coastal Model



Lynch et al. (Oceanography 2001)



Major Problems in Coastal Modeling

- *(1) Discretization*
- *(2) Sigma Error*
- *(3) Difference Schemes*
- *(4) POM Capability*
- *(5) Air-Ocean Coupling*
- *(6) Severe Weather Effect*
- *(7) Velocity Data Assimilation*
- *(8) Turbulence/Wave Effects*

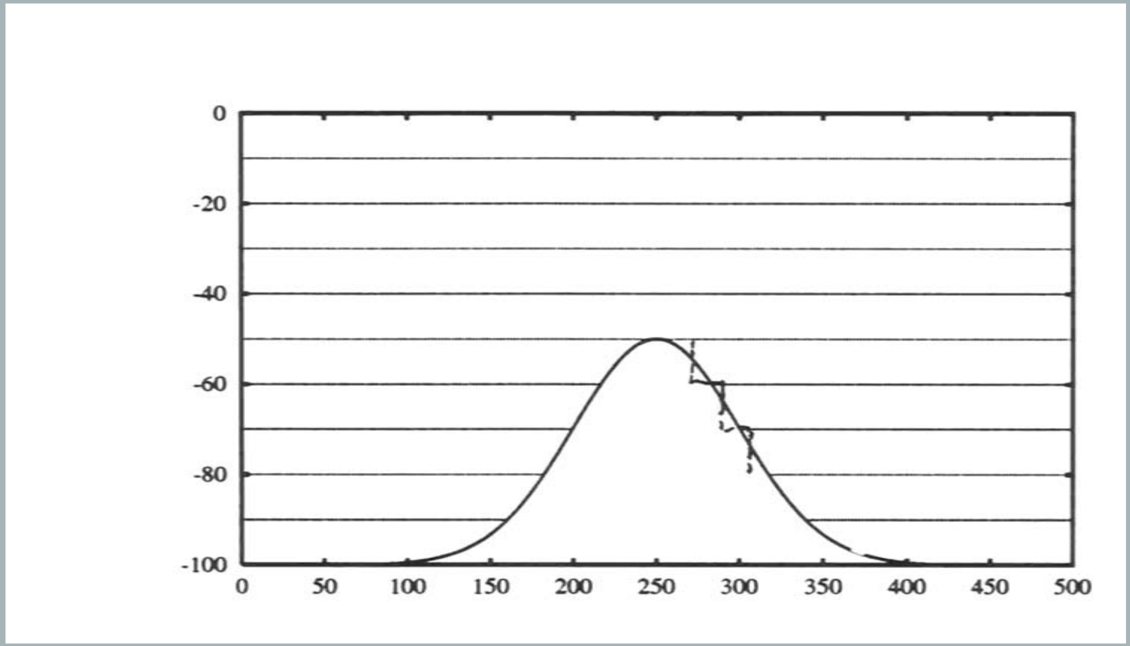
(1) Discretization

Diversity in Discretization

- *Finite Differences*
  - *Z – coordinate (MOM, ...)*
  - *σ - coordinate (POM, COHERENS, etc…)*
  - *s- coordinate (SCRUM, ROMS ...)*
  - *Layered/Isopycnal coordinates (NLOM, MICOM, ...)*
- *Finite Elements*

Z-Coordiante

- *Note "staircase" topography representation, normally with no-slip conditions*

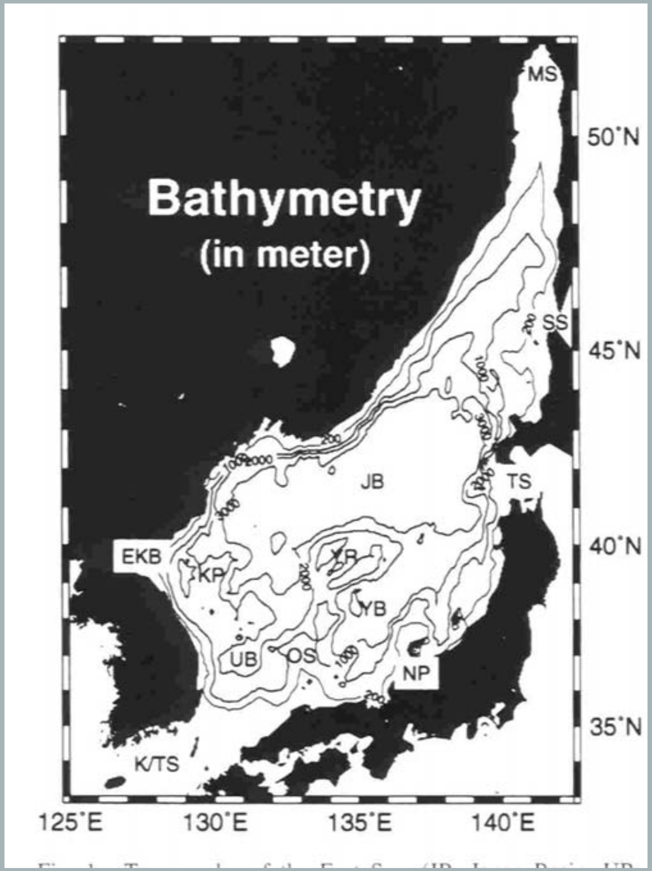


Problems of the "Staircase Presentation"

- *Difficult in simulating coastal flow.*
- *Example: Japan/East Sea (JES) Simulation (Kim and Yoon, 1998 JO)*

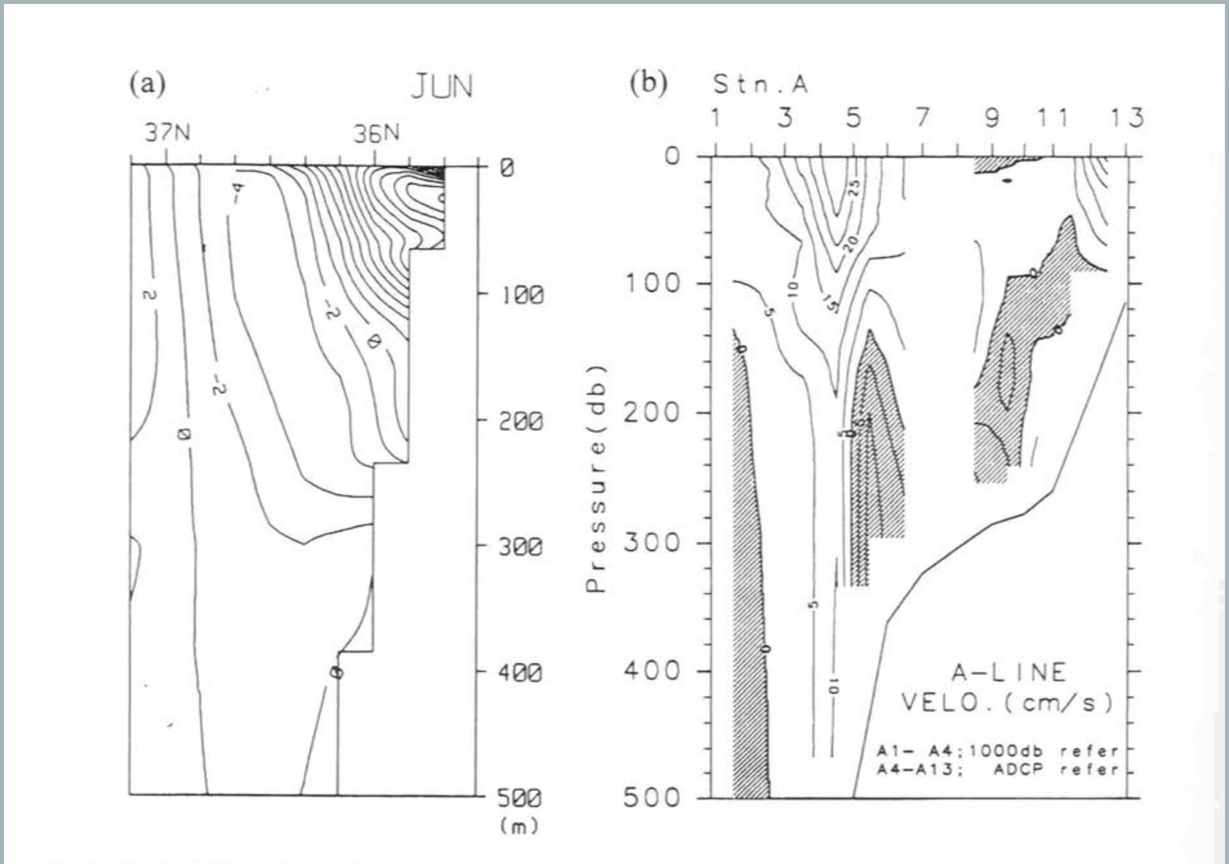
JES Circulation Model Using MOM (Kim & Yoon, 1998)

- *1/6 deg resolution*
- *19 vertical level*
- *Monthly mean wind stress (Na et al. 1992)*
- *Monthly mean heat flux (Haney type)*



Problem in Simulating Coastal Currents

- *Model* *Observation*



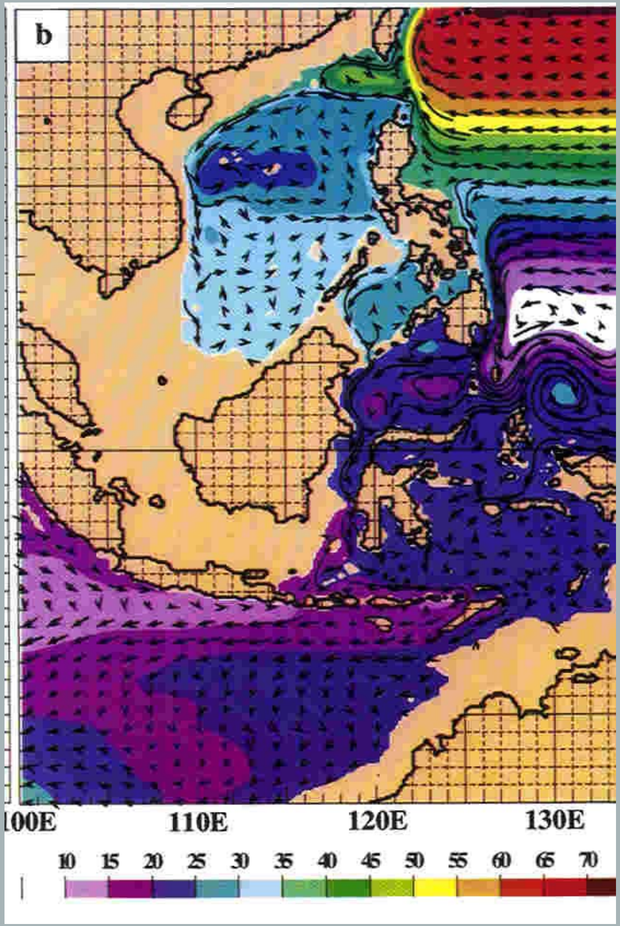
Layered/Isopycnal Coordinates

- *Pro*
  - *Horizontal mixing is exactly along the surfaces of constant potential density*
  - *Avoids inconsistencies between vertical and horizontal transport terms*

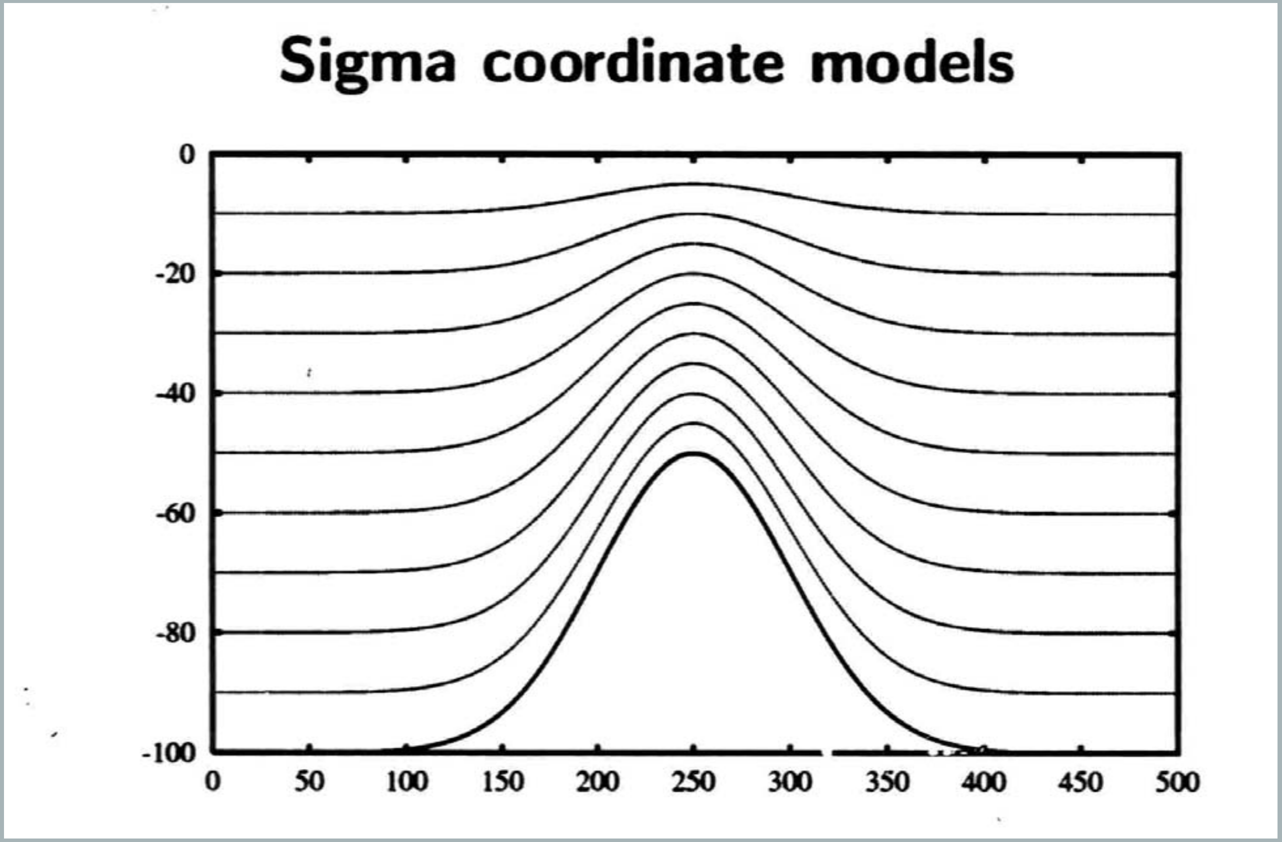
- *Con*
  - *It requires an evident layered structure (not suitable for shelf circulation*
  - *Some difficulty in modeling detrainment of ocean mixed layer*

Layered/Isopycnal Coordinates

- *(Metzger and Hurlburt 1996, JGR)*
- *1/8º, 6 layer with realistic bottom topography*
- *Not applicable to simulating shelf circulation*



Sigma Coordinate Models



Sigma Coordinates

- *Pro*
  - *Realistic Bottom Topography*
  - *Applicable to Shelf and Estuarine Circulation*
- *Con*
  - *Horizontal Pressure gradient Error*
  - *High Vertical Resolution in Shallow Water (Shelf) and Low Resolution in Deep Water*

Horizontal Diffusion

- *The second and fourth terms in the righthand side are neglected.*

Diffusion of tracer fields in Sigma coordinates

The horizontal mixing becomes

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_z = \left(\frac{\partial^2 T}{\partial x^2}\right)_\sigma + \frac{\partial}{\partial x}\left(\sigma_x \frac{\partial T}{\partial \sigma}\right)_\sigma + \frac{\partial}{\partial \sigma}\left(\sigma_x \left.\frac{\partial T}{\partial x}\right|_\sigma\right) + \sigma_x^2 \frac{\partial^2 T}{\partial \sigma^2},$$

where $\sigma_x = -\frac{\sigma}{h}\frac{\partial h}{\partial x}$.

(2) Sigma Error

Pressure Gradient Error

Uses a coordinate system which is scaled with the depth

$$\sigma = \frac{z - \zeta}{H + \zeta}.$$

Makes use of coordinate surfaces which are located below the bottom depth in Level models.

Aspect ratio	Ocean	Atmosphere
$\frac{\Delta h_{\text{topography}}}{H_{\text{depth}}}$	$\sim \mathcal{O}(1)$	$\ll \mathcal{O}(1)$

- Atmosphere: Hybrid (σ, p) models are common.
- Ocean: ρ and σ surfaces may intersect at large angle.

Pressure Gradient Error

Pressure gradient in Sigma coordinates

$$\left.\frac{\partial P}{\partial x}\right|_{z=\mathrm{const}} = \left.\frac{\partial P}{\partial x}\right|_{\sigma=\mathrm{const}} - \frac{\sigma}{h}\frac{\partial h}{\partial x}\frac{\partial P}{\partial \sigma}$$

(1) Pressure grad along σ-surfaces

(2) Correction for vertical component in (1)

In case of large slopes:

$$\mathcal{O}(2) = \mathcal{O}(1)$$

$$(1) + (2) \ll (1) \vee (2)$$

Thus, truncation errors may be significant.

Seamount Test Case

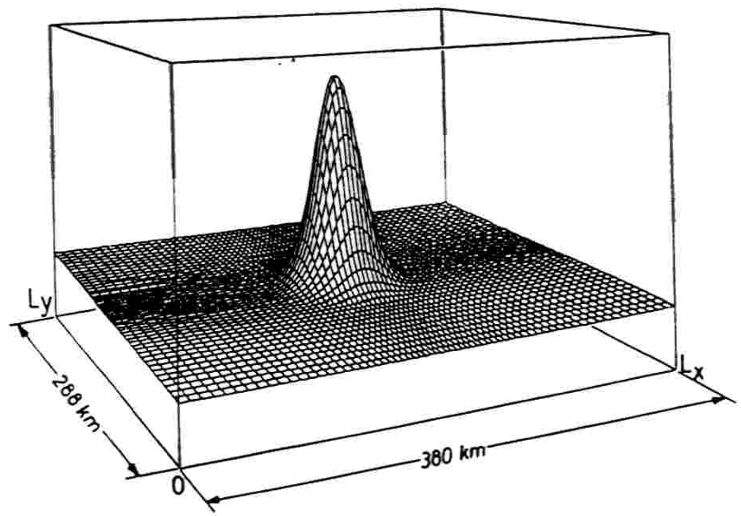


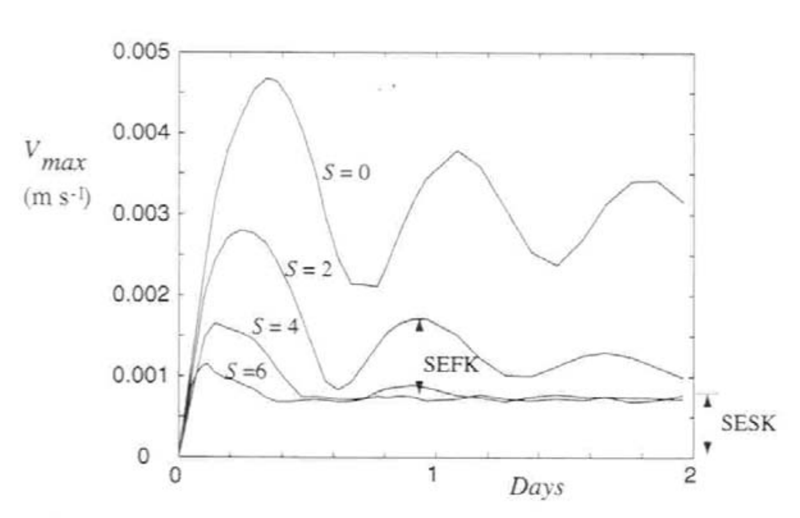
FIG. 1. The seamount geometry. The grid is stretched so that the resoluton is highest at the center (after Beckman and Haidvogel 1993).

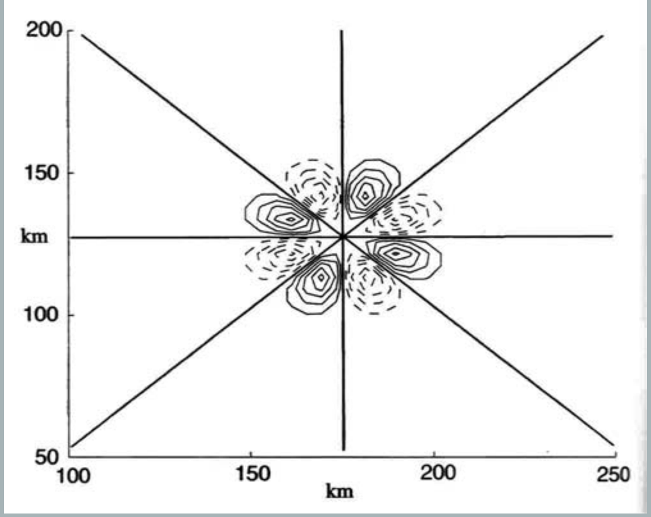
$$S \equiv \frac{N_o H_o}{f_o L} = \frac{(g H_o \Delta_z \rho / \rho_o)^{1/2}}{f_o L},$$

Two Kinds of Sigma Errors (Mellor et al. 1998, JTECH)

- *First Kind (SEFK):*
  Horizontal Density Gradient
  Oscillatory Decaying

- *Second Kind (SESF)*
  - *Vorticity Error*

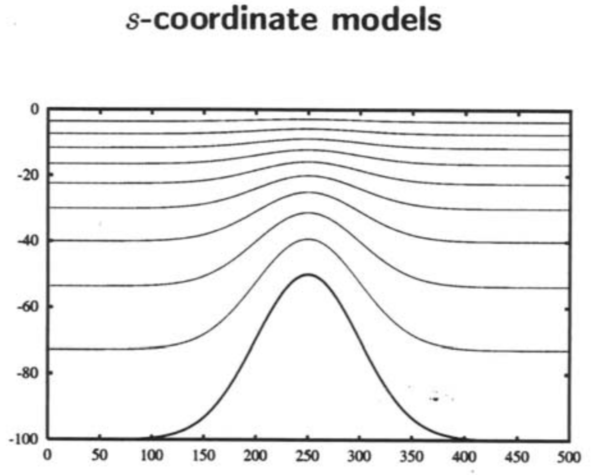




Reduction of Sigma Error

- *Smoothing topography*
- *Subtracting horizontally averaged density field*
- *Using generalized topography-following coordinate system (e.g., S-coordinates in ROMS)*
- *Using high-order difference schemes*

S-Coordinate
Generalized Topography-Following Coordinates (Song & Haidvogel, 1994)



Uses a general vertical coordinate scaled with the depth using a nonlinear transformation. More flexible than traditional sigma coordintes (e.g., allows for high resolution in bottom and surface boundary layers).

$$\left.\frac{\partial p}{\partial x}\right)_z = \left.\frac{\partial p}{\partial x}\right)_{z=\zeta} + \frac{g}{\rho_0}\int_s^0 \left\{\frac{\partial z}{\partial s'}\frac{\partial \rho}{\partial x} - \frac{\partial z}{\partial x}\frac{\partial \rho}{\partial s'}\right\} ds',$$

Error Analysis (S-Coordinate)

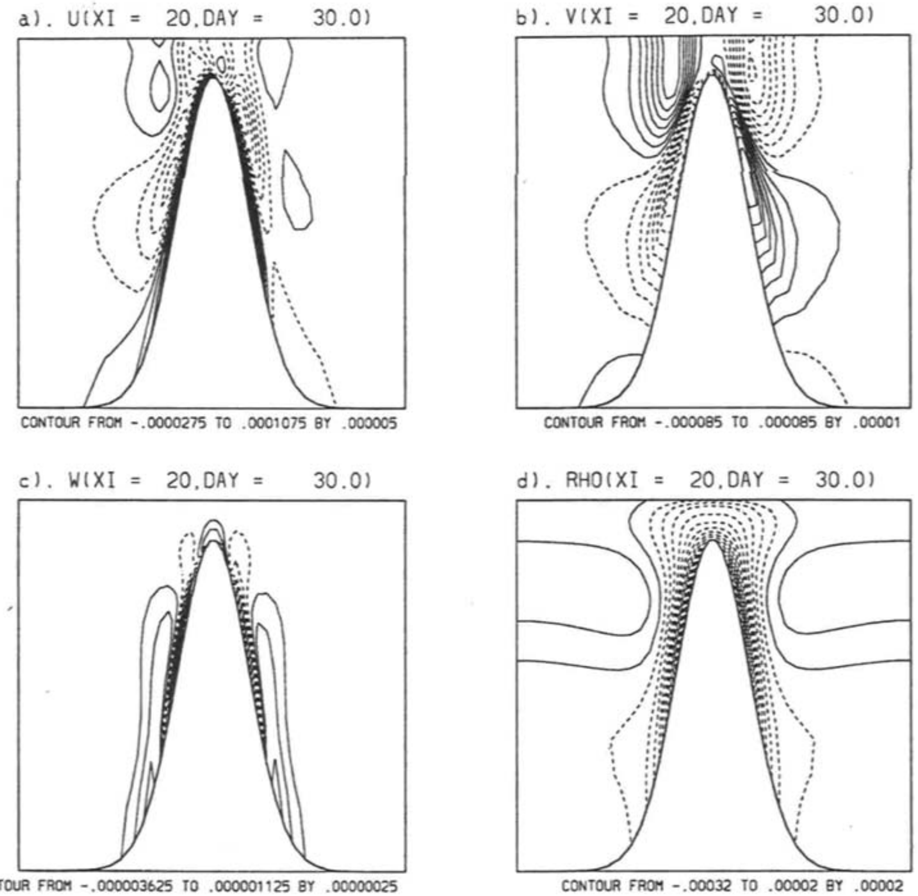
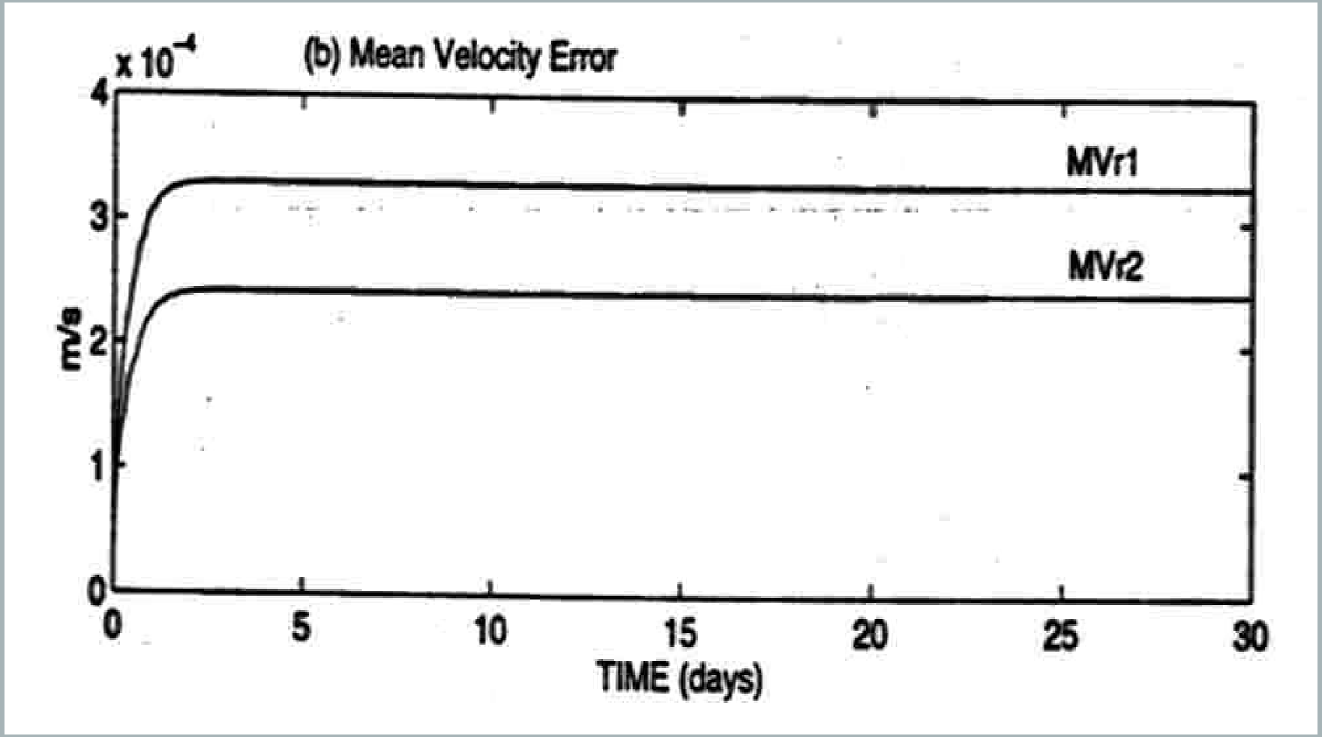


FIG. 1. Cross sections through the center of the seamount: (a) the alongshore velocity; (b) the cross-shore velocity; (c) the vertical velocity; (d) the density change relative to the initial conditions. Units are in m s^{-1} for u, v, w, and in kg m^{-3} for ρ.

Error Evolution (S-coordinate)

- *Radius of Seamount: r1 = 40 km, r2 =80 km*



High-Order Schemes

- *Ordinary Five-Point Sixth-Order Scheme (Chu and Fan, 1997 JPO)*
- *Three-Point Sixth-Order Combined Compact Difference (CCD) Scheme (Chu and Fan, 1998 JCP)*
- *Three-Point Sixth-Order Nonuniform CCD Scheme (Chu and Fan, 1999, JCP)*
- *Three-Point Sixth-Order Staggered CCD Scheme (Chu and Fan, 2000, Math. & Comp. Modeling)*
- *Accuracy Progressive Sixth-Order Scheme (Chu and Fan, 2001, JTECH)*

(3) Difference Schemes

Why do we need high-order schemes?

- *(1) Most ocean circulation models are hydrostatic.*

- *(2) If keeping the same physics, the grid space (Δx) should be larger than certain criterion such that the aspect ratio*

$$\delta = H / \Delta x << 1$$

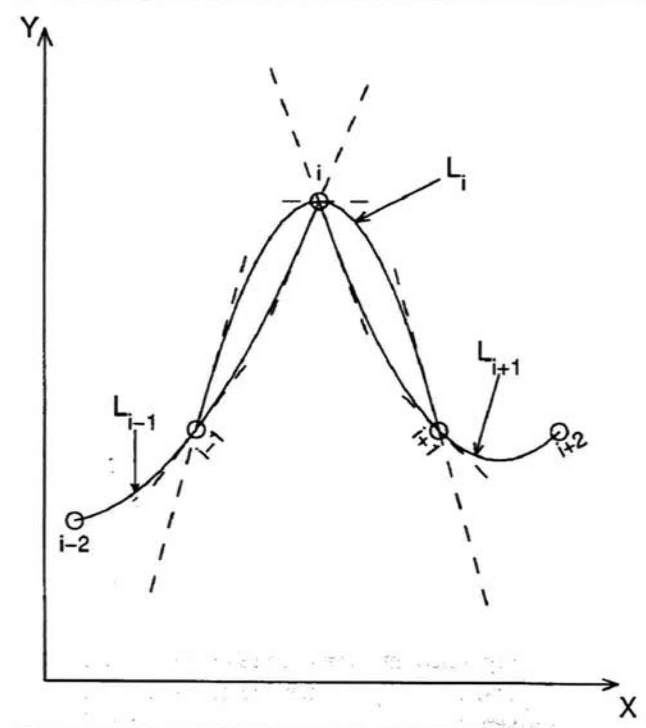
A Hidden Problem in Second Order Central Difference Scheme

$$\phi_i' = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$$

$$\phi_i'' = \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta x^2}$$

- *Local Hermitian Polynomials*

Discontinuity of the first derivatives of the Lagrangian Polynomials at each grid point.



- *Both Φ' and Φ'' are not continuous at each grid point. This may cause some problems.*

Three-Point Sixth-Order Scheme

$$\left(\frac{\delta f}{\delta x}\right)_i + \alpha_1 \left(\left(\frac{\delta f}{\delta x}\right)_{i+1} + \left(\frac{\delta f}{\delta x}\right)_{i-1}\right) + \beta_1 h \left(\left(\frac{\delta^2 f}{\delta x^2}\right)_{i+1} - \left(\frac{\delta^2 f}{\delta x^2}\right)_{i-1}\right) + \ldots$$

$$= \frac{a_1}{2h}\left(f_{i+1} - f_{i-1}\right)$$

$$\left(\frac{\delta^2 f}{\delta x^2}\right)_i + \alpha_2 \left(\left(\frac{\delta^2 f}{\delta x^2}\right)_{i+1} + \left(\frac{\delta^2 f}{\delta x^2}\right)_{i-1}\right) + \beta_2 \frac{1}{2h} \left(\left(\frac{\delta f}{\delta x}\right)_{i+1} - \left(\frac{\delta f}{\delta x}\right)_{i-1}\right) + \ldots$$

$$= \frac{a_2}{h^2}\left(f_{i+1} - 2f_i + f_{i-1}\right)$$

Three-Point Sixth Order CCD Schemes

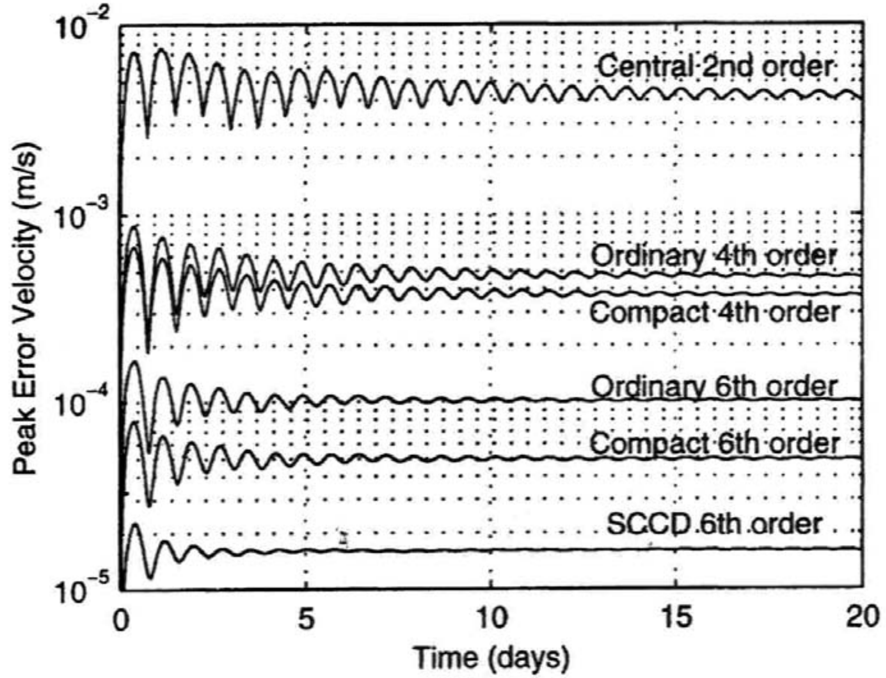
- *Existence of Global Hermitian Polynomials*
- *First Derivative Continuous*

$$H_i'(x_i) = H_{i-1}'(x_i) = H_{i+1}'(x_i) = \left(\frac{\delta f}{\delta x}\right)_i$$

- *Second Derivative Continuous*

$$H_i''(x_i) = H_{i-1}''(x_i) = H_{i+1}''(x_i) = \left(\frac{\delta^2 f}{\delta x^2}\right)_i$$

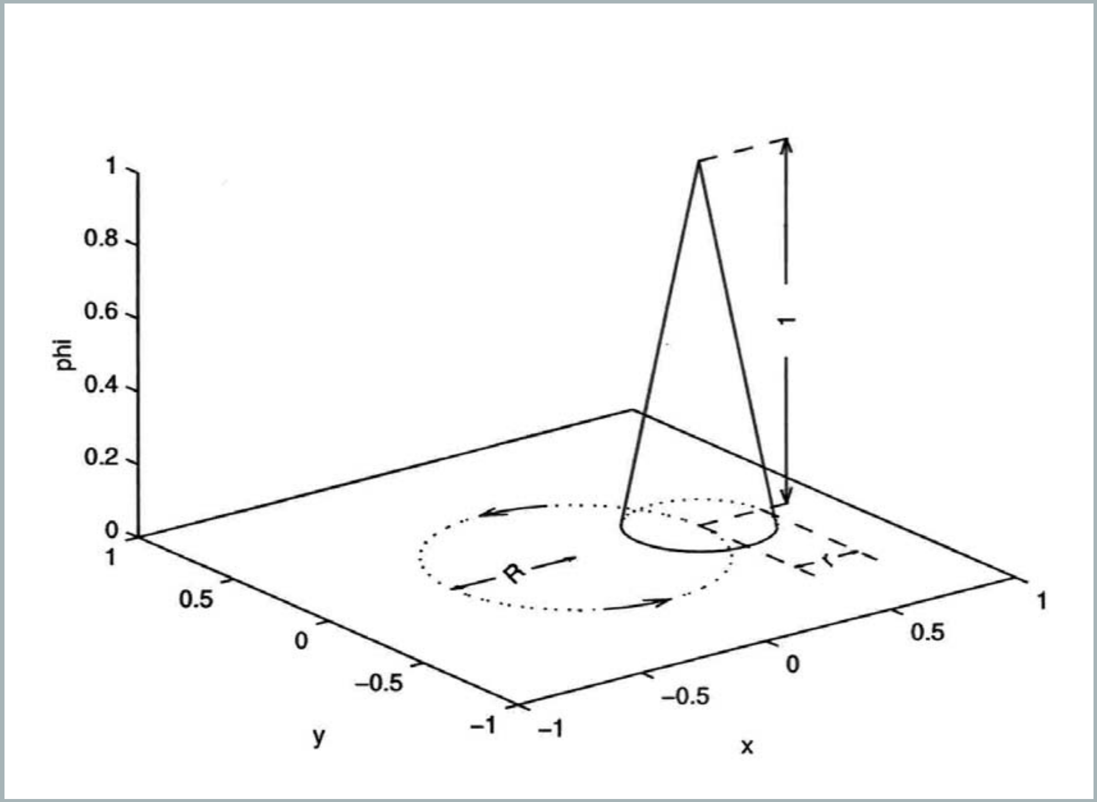
Error Reduction Using CCD Schemes (Seamount)



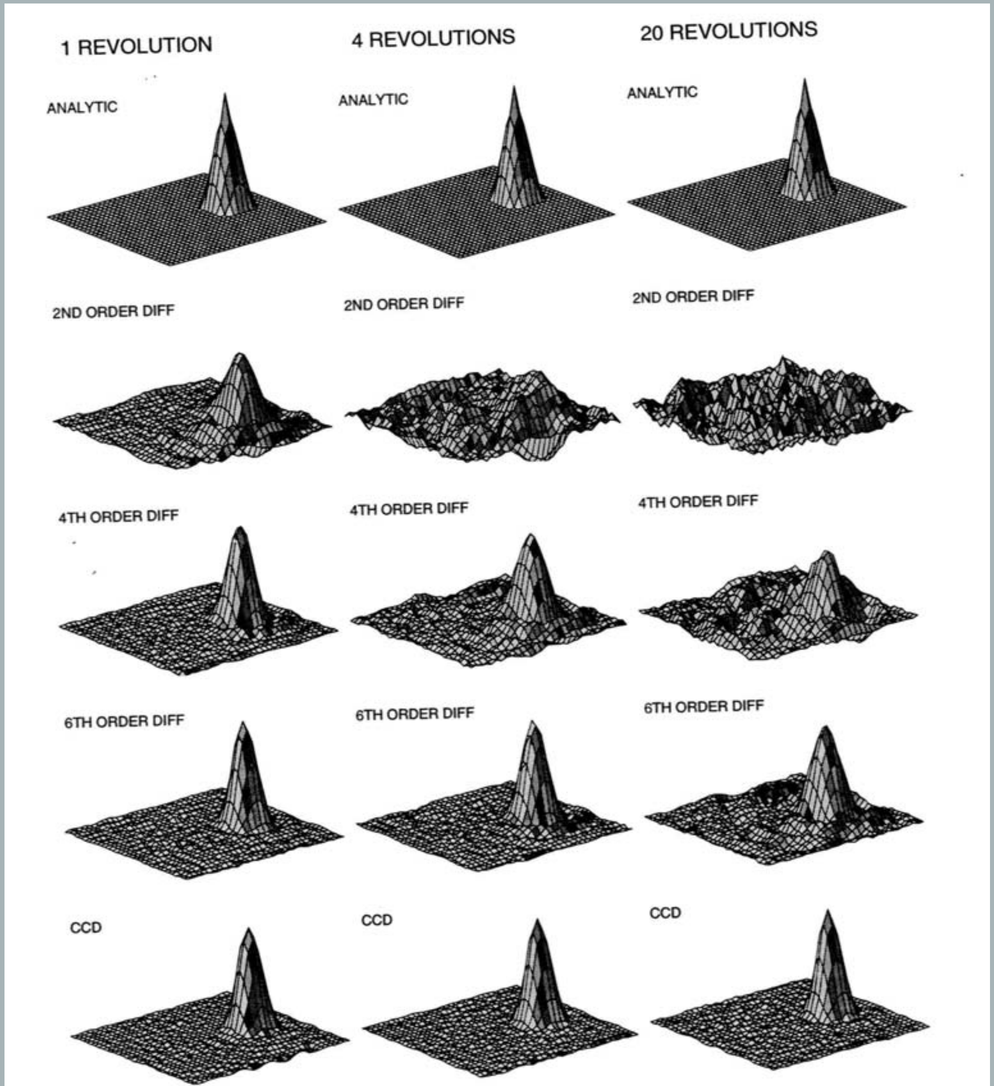
(a) Peak error velocity.

Figure 11. Peak error velocity propagation in 20 days for the SCCD, second-order, fourth-order, sixth-order ordinary, and sixth-order compact schemes (all staggered). The formulas for the compact schemes compared are listed in the Appendix.

Rotating Cone for Testing Various Schemes



Accuracy Comparison

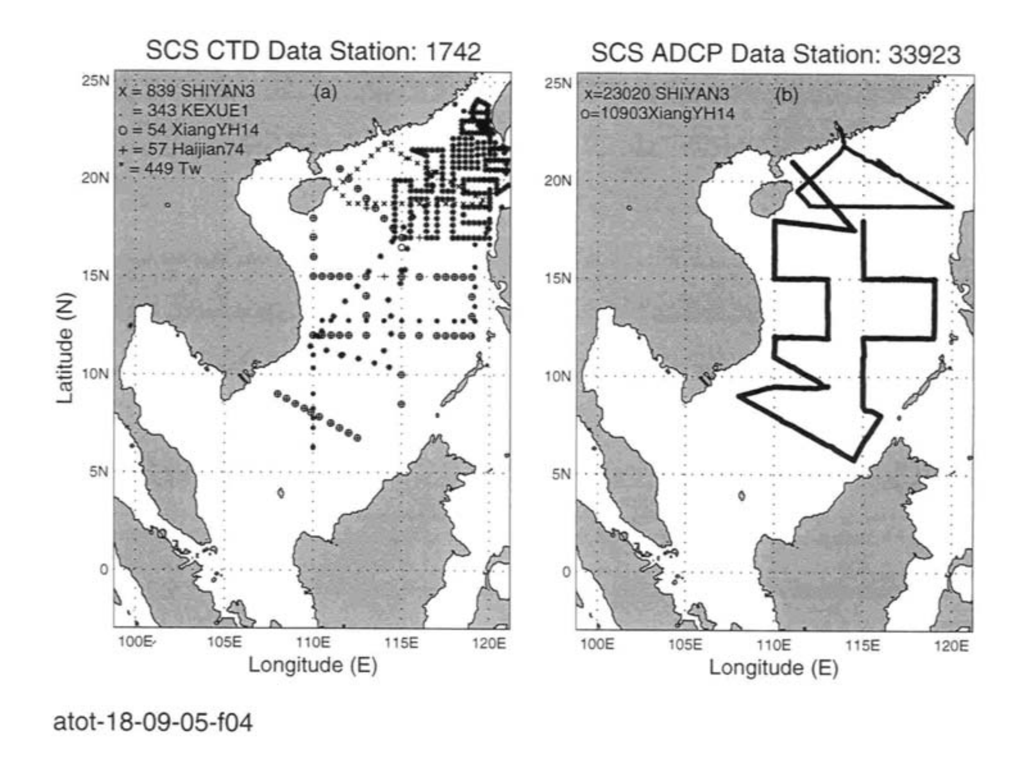


(4) POM Capability

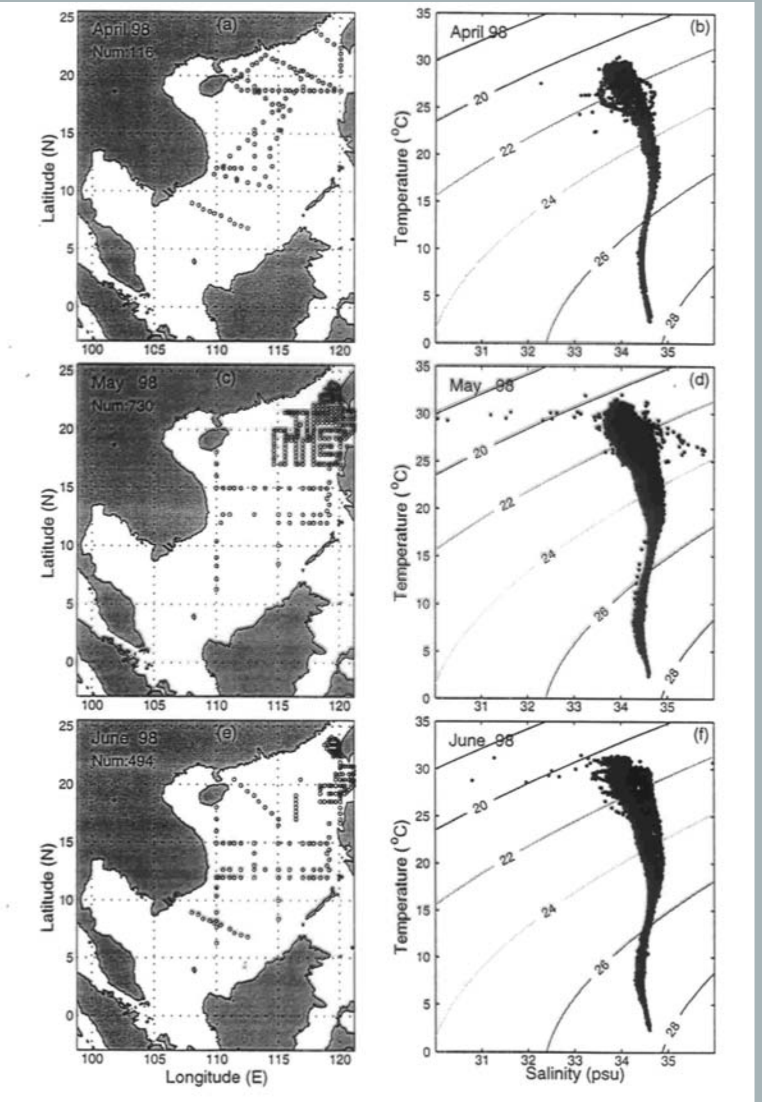
Chu et al 2001, JTECH

Evaluation of POM Using the South China Sea Monsoon Experiment (SCSMEX) Data

- *IOP (April – June 1998)*



T-S Diagram from SCSMEX Observations



Two Step Initialization of POM

- *(1) Spin-up*
  - *Initial conditions: annual mean (T,S) + zero velocity*
  - *Climatological annual mean winds + Restoring type thermohaline flux (2 years)*
- *(2) Climatological Forcing*
  - *Monthly mean winds + thermohaline fluxes from COADS (3 years) to 1 April*
- *(3) The final state of the previous step is the initial state of the following step*
- *(4) Synoptic Forcing*
  - *NCEP Winds and Fluxes: April 1 to June 30, 1998 (3 Months)*

Two Types of Model Integration

- *(1) MD1:*
  - *Without Data Assimilation*
  - *Hindcast Period: April-June 1998 (3 Months)*
- *(2) MD2:*
  - *With Daily SCSMEX-CTD Data Assimilation*
  - *Hindcast Period:*
    - *May 1998: No data Assimilation in May*
    - *June 1998: No data Assimilation in June*

Skill-Score

- *Model-Data Difference*

$$\Delta\psi(x_i, y_j, z, t) = \psi_m(x_i, y_j, z, t) - \psi_o(x_i, y_j, z, t).$$

-
- *Mean Square Error*

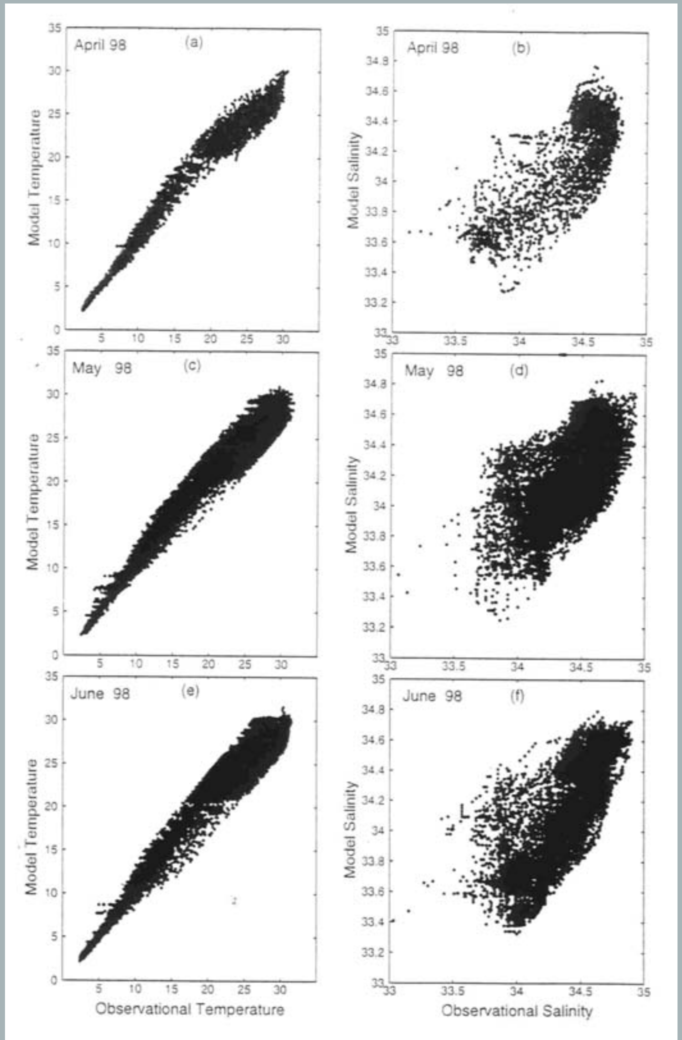
$$\mathrm{MSE}(z,t) = \sum_i \sum_j \frac{1}{N} \left[\Delta\psi(x_i, y_j, z, t)\right]^2$$

- *Skill-Score (SS)*

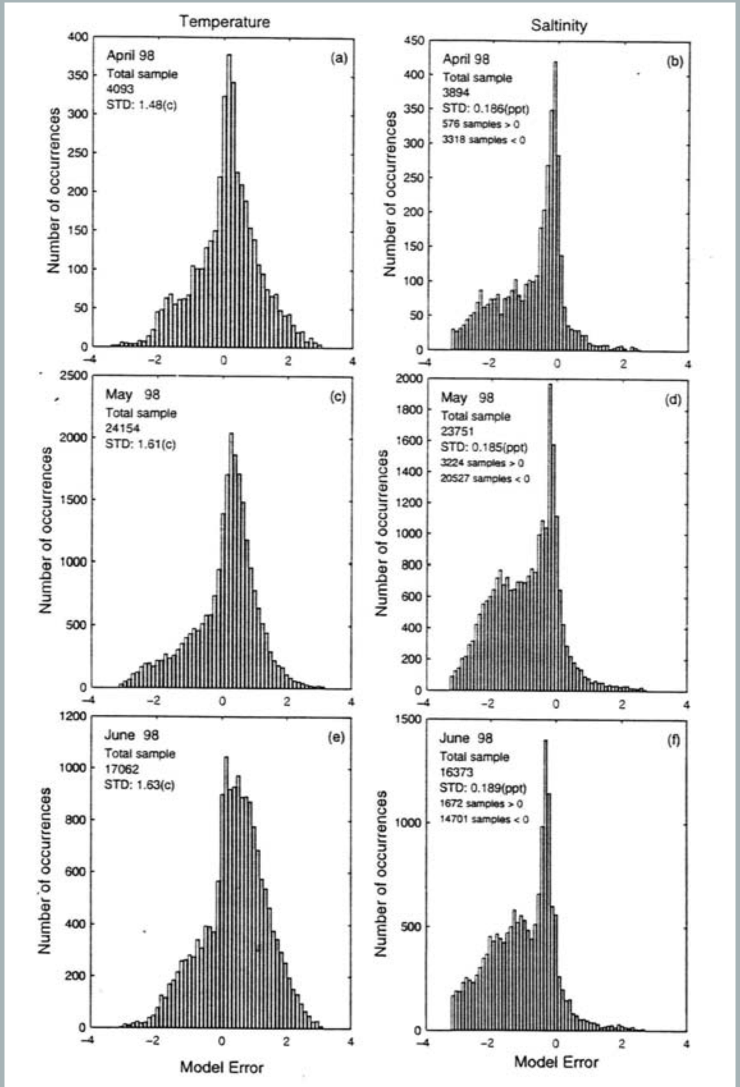
$$SS = 1 - \frac{\mathrm{MSE}(m,o)}{\mathrm{MSE}(c,o)},$$

- *SS > 0, Model has capability*

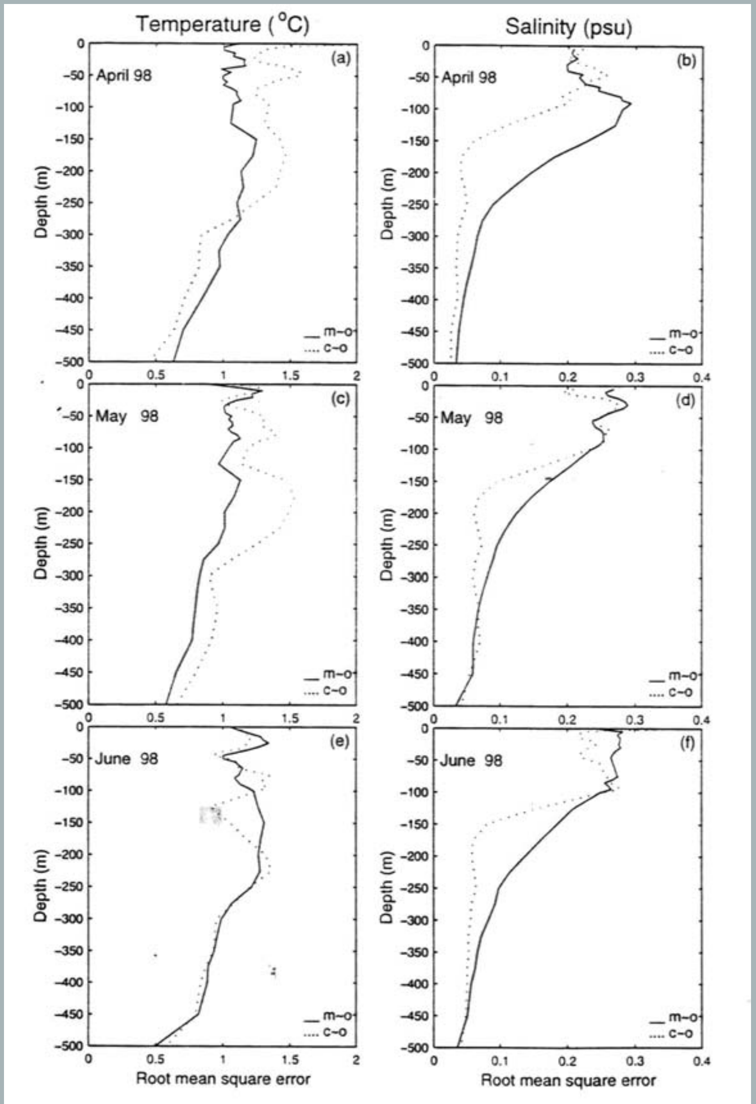
Scatter Diagrams Between Model and Observation (MD1)



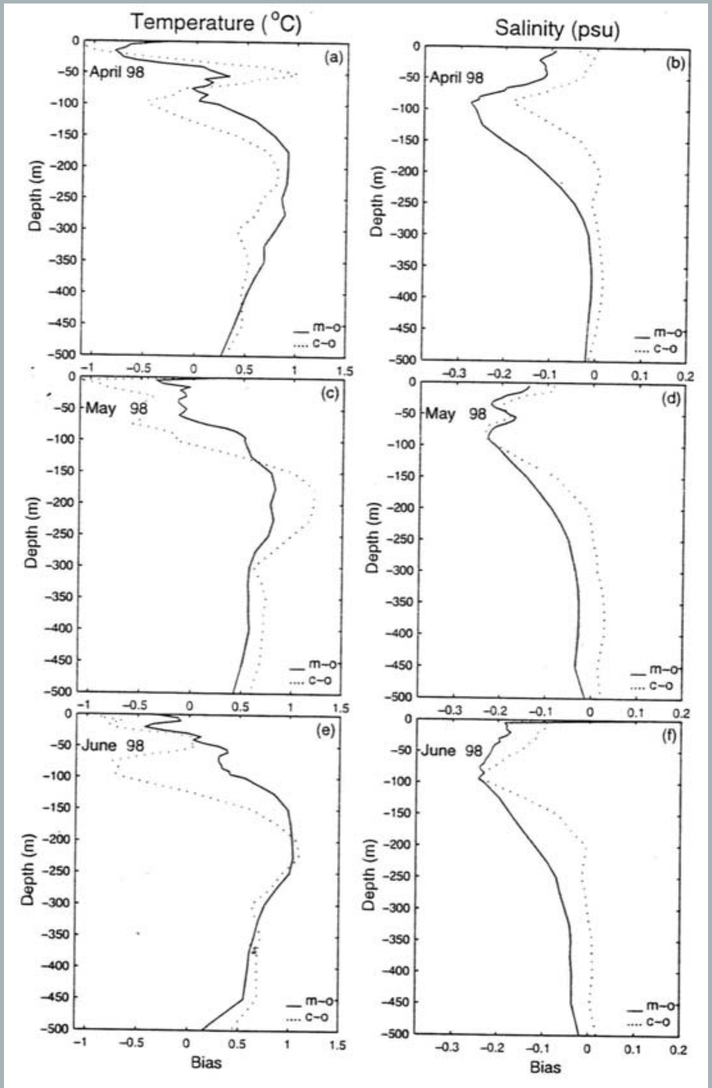
Histograms of (Model – Obs) for MD1



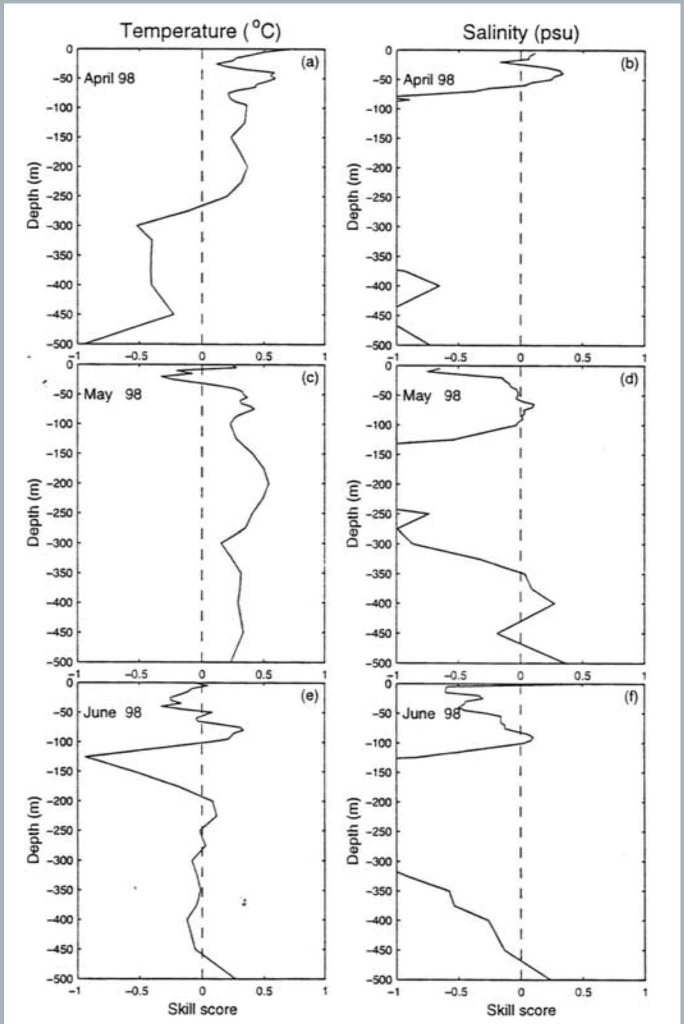
RMS Error for MD1 (No Assimilation)



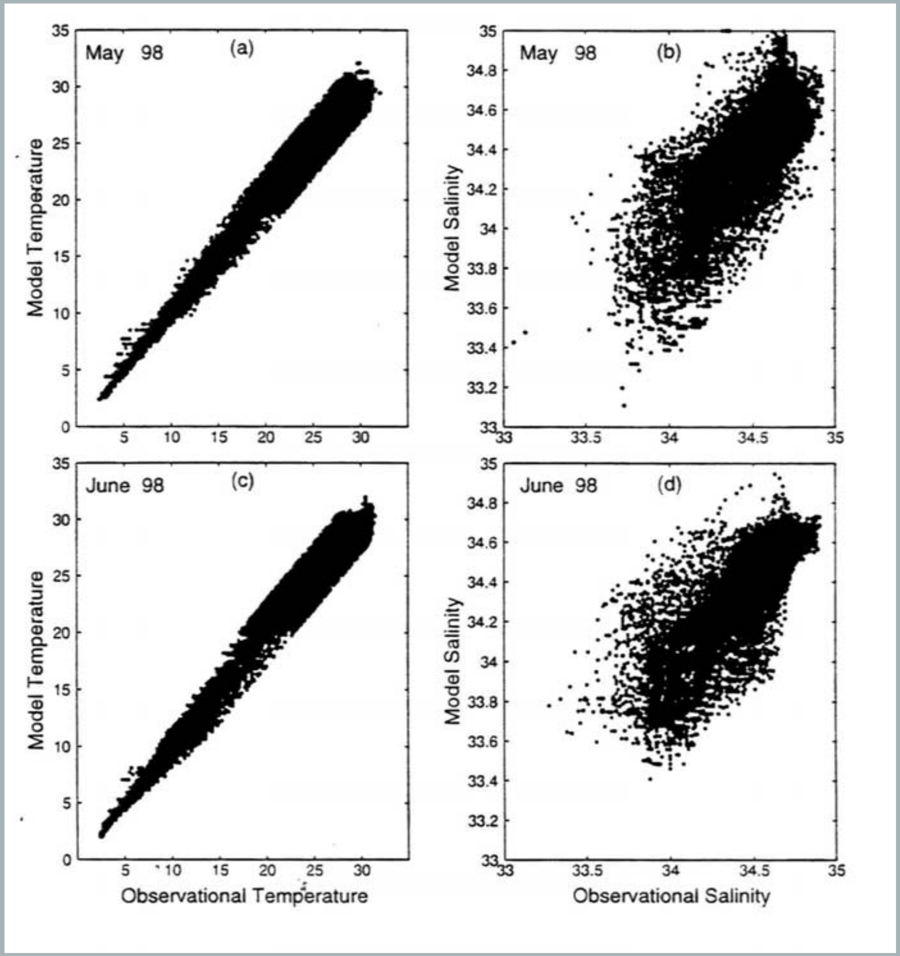
Bias for MD1 (No Assimilation)



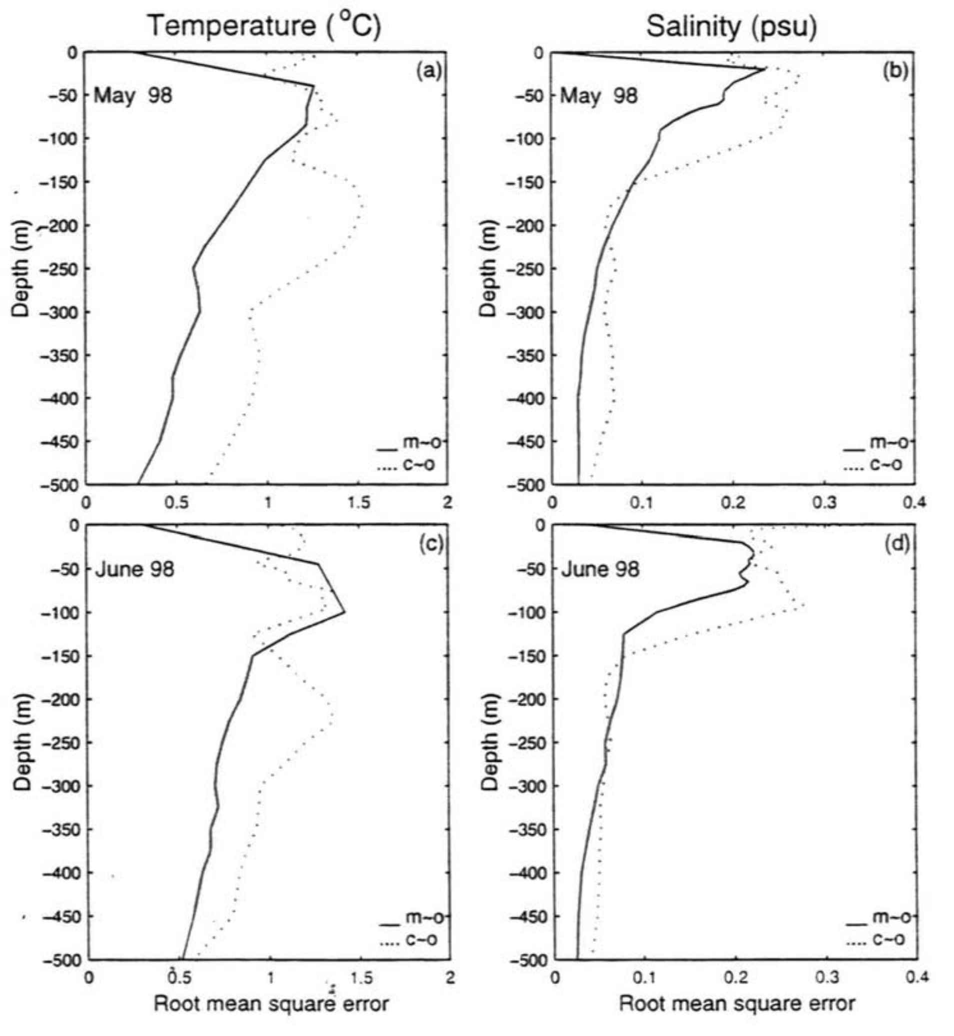
Skill-Score for MD1 (No Assimilation)



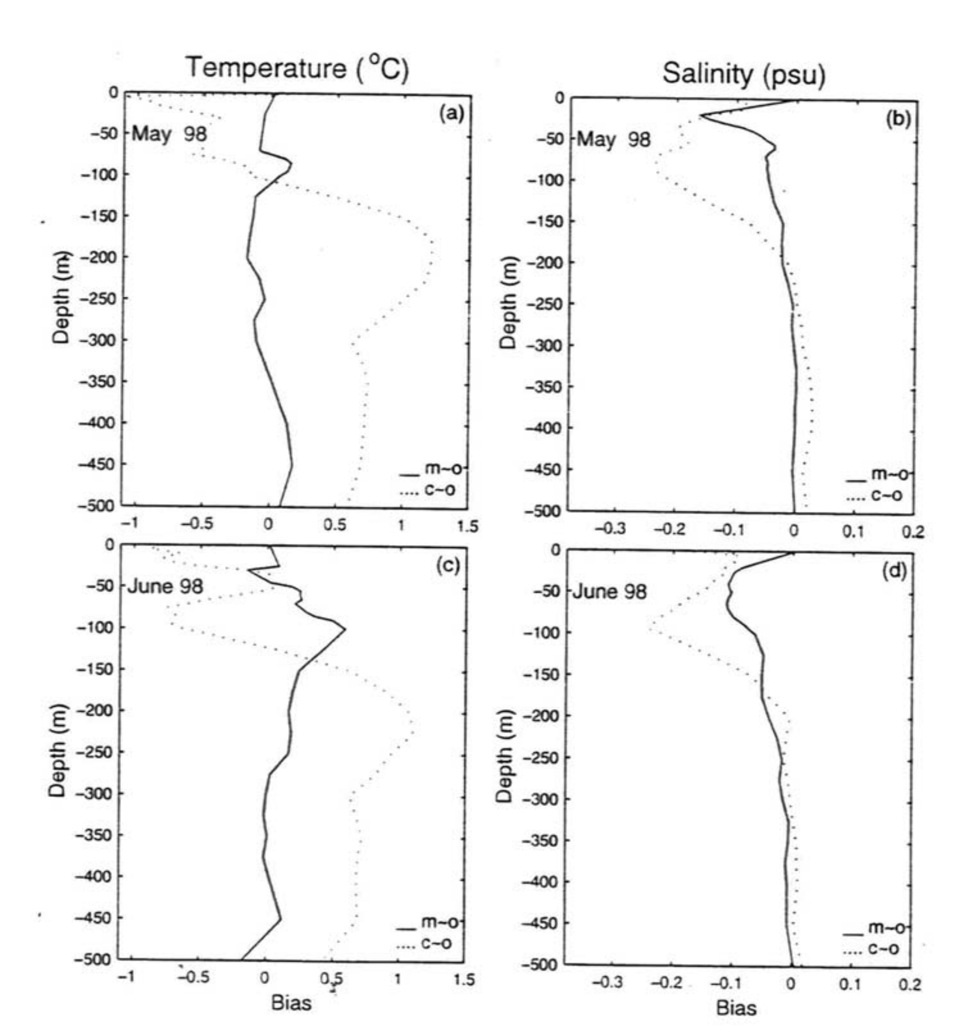
Scatter Diagrams for MD2 (with Assimilation)



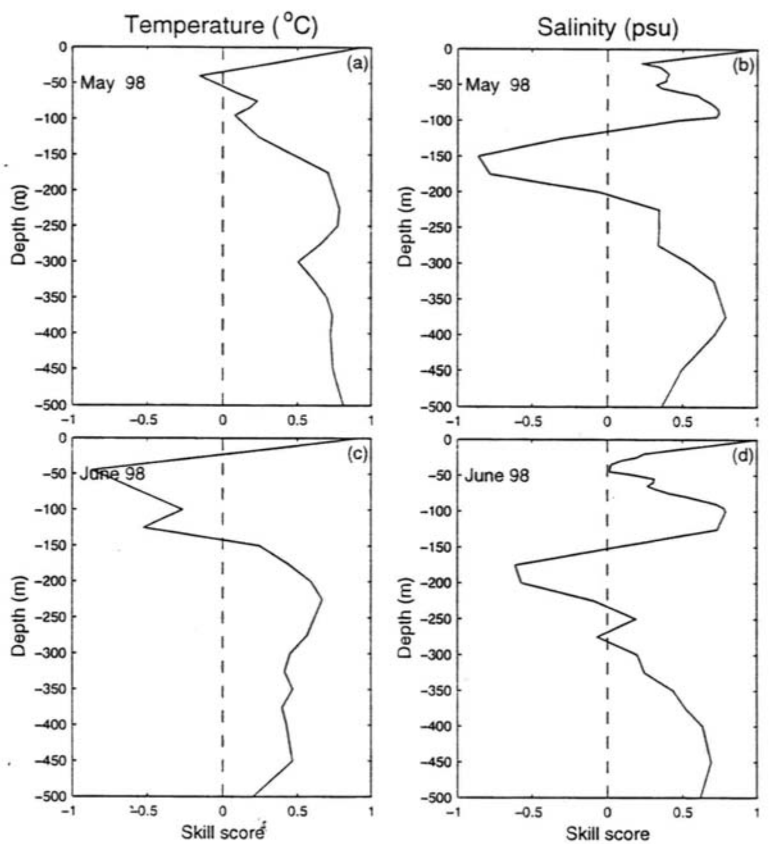
RMS Error for MD2 (with Assimilation)



Bias for MD2 (with Assimilation)



Skill-Score for MD2 (with Assimilation)



Comments

- *(1) POM-SCS has synoptic flux forcing.*
- *(2) Without data assimilation, it has capability to predict temperature, but not salinity.*
- *(3) With data assimilation, it has capability to predict salinity.*

(5) Air-Ocean Coupling

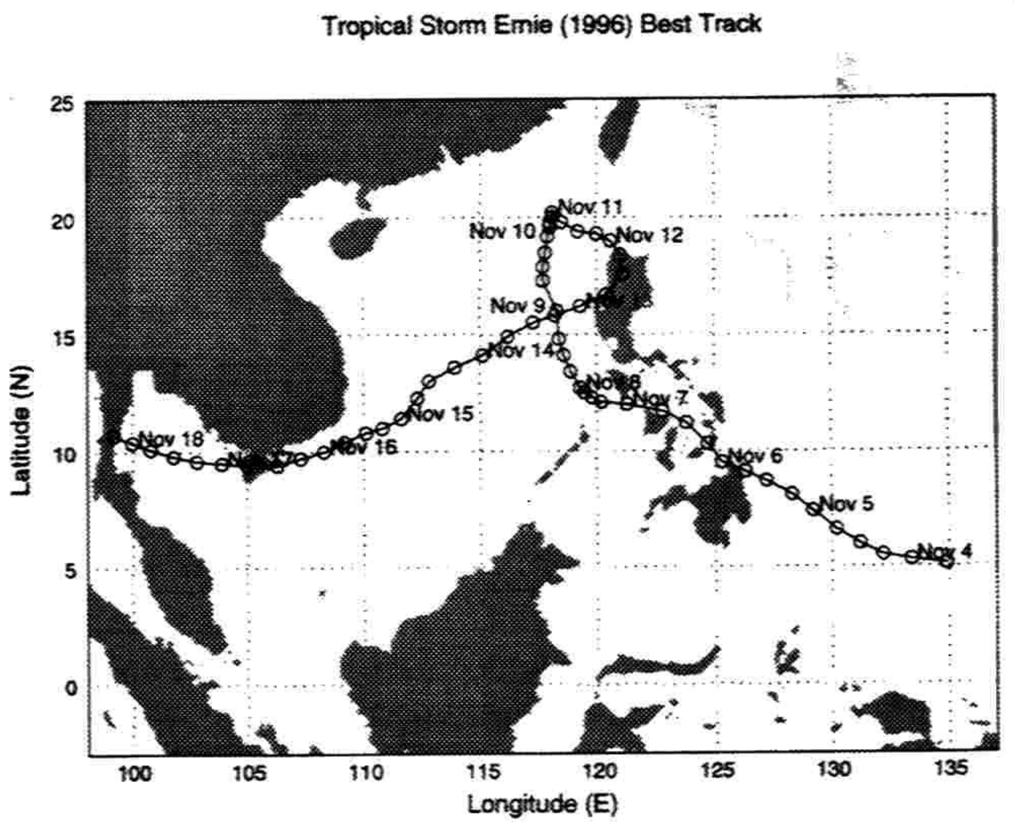
- *Coastal Atmosphere-Ocean Coupled System (CAOCS) for East Asian Marginal Sea (EAMS) Prediction*
- *Chu et al. (1999, JO)*

Necessity for Air-Ocean Coupling

- *(1) Sparse Meteorological Observation over Ocean*
- *(2) Uncertain Surface Fluxes*
- *(3)   Nowcast/Forecast*

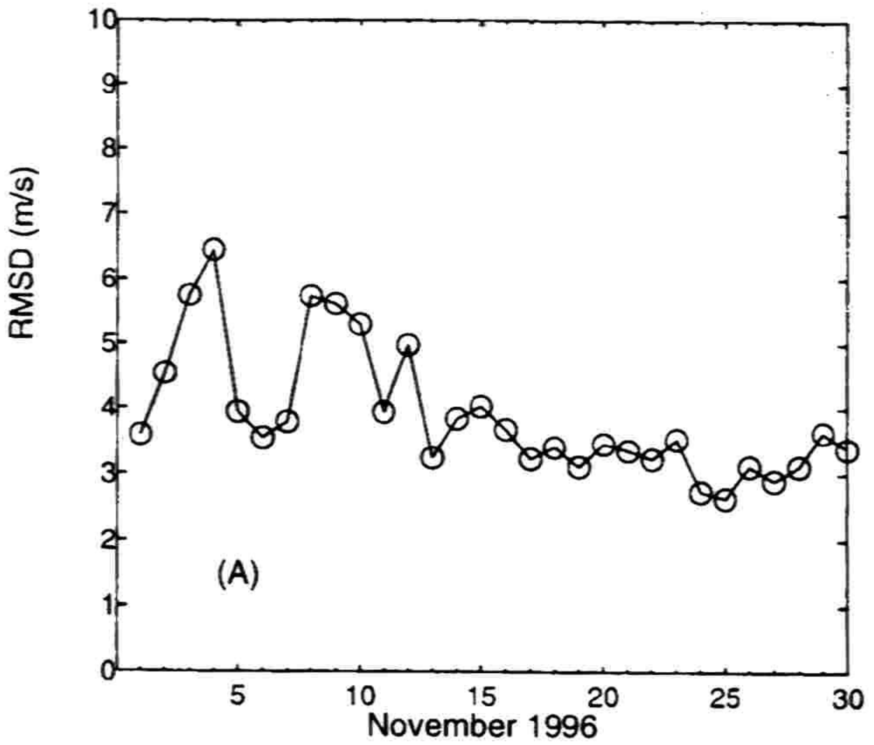
Uncertain Atmospheric Forcing

The track of the tropical cyclone Ernie 4-18 November, 1996 (from Chu et al., 1998).



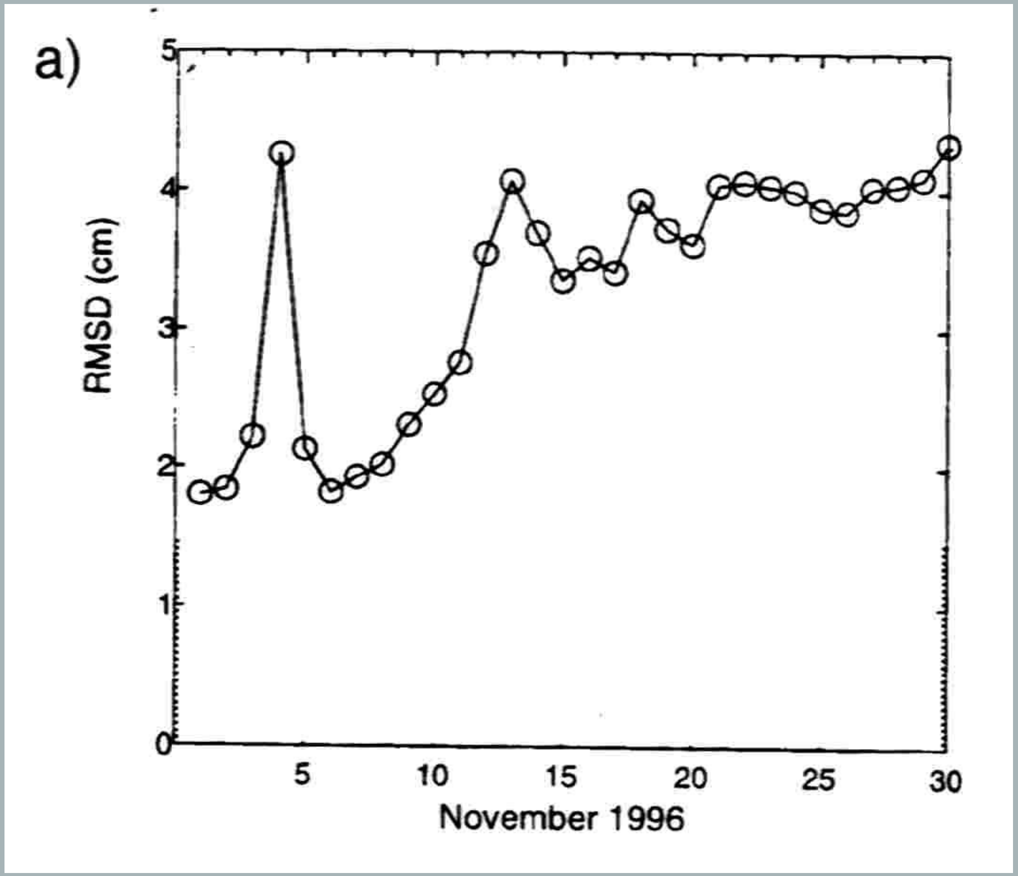
RMS Difference Between NSCAT and NCEP Winds

Temporally varying root-mean-square difference between daily mean NSCAT and NCEP winds over the whole South China Sea (from Chu et al., 1998).



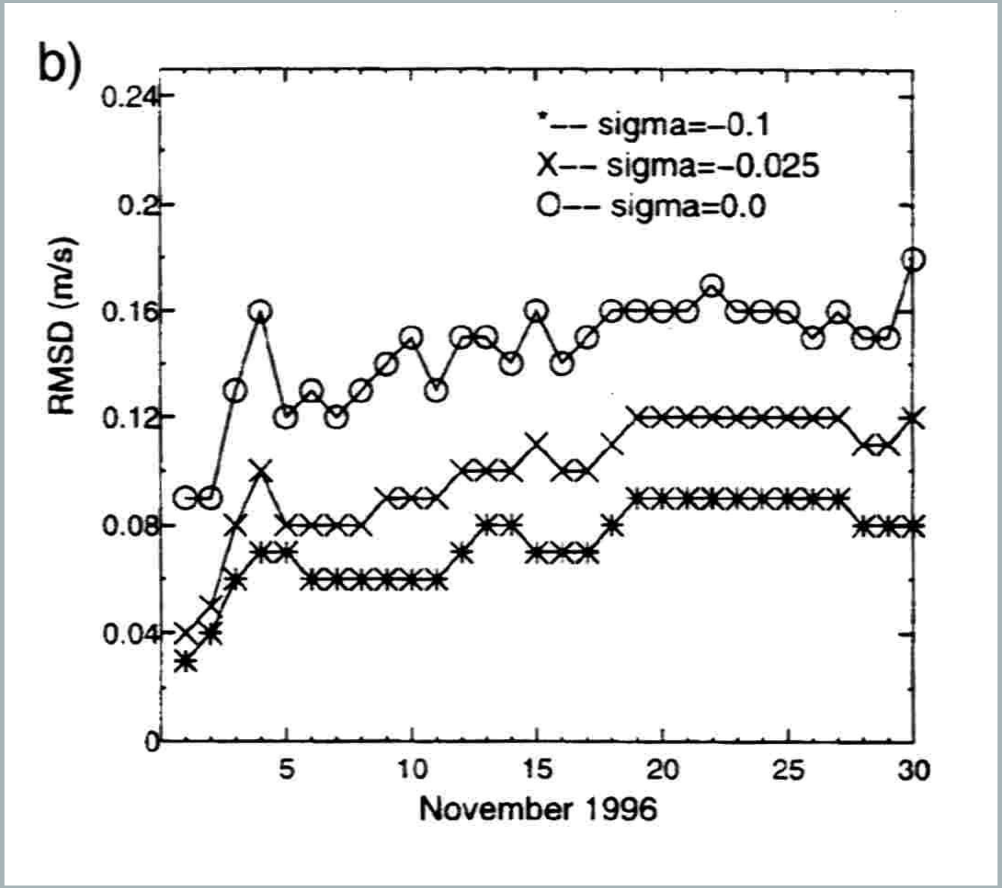
Temporally Varying RMS Difference Between POM Model Results Under the Two Wind Forcing (Chu et al. 1998, JGR)

- *Surface elevation*



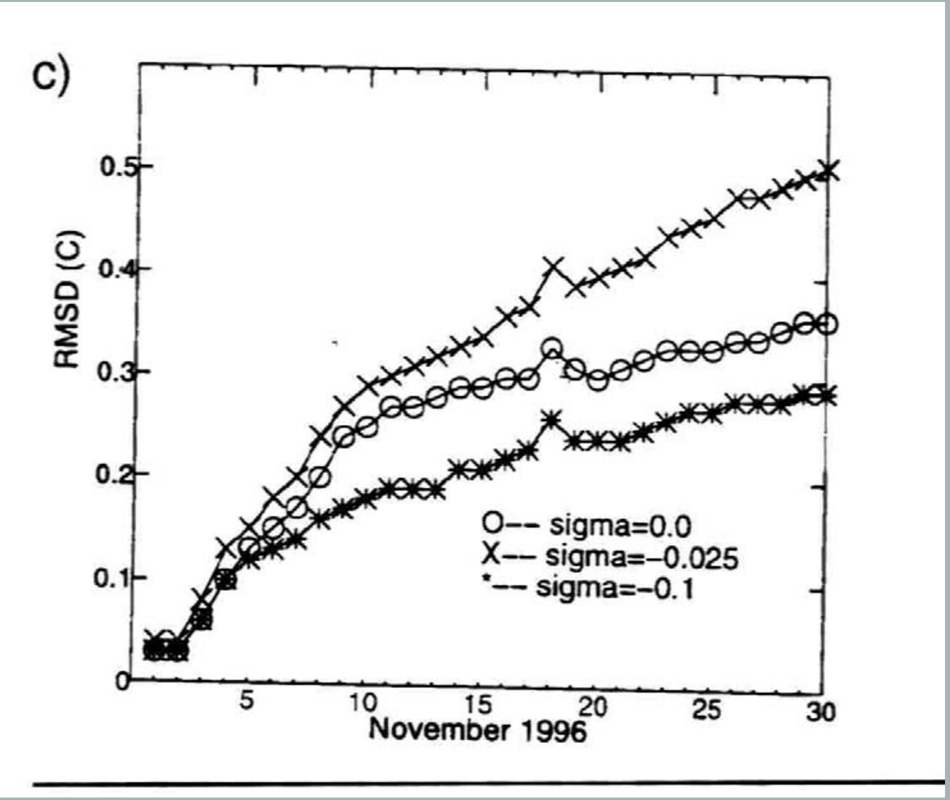
Temporally Varying RMS Difference Between POM Model Results Under the Two Wind Forcing (Chu et al. 1998, JGR)

- *Velocity*



Temporally Varying RMS Difference Between POM Model Results Under the Two Wind Forcing (Chu et al. 1998, JGR)

- *Temperature*



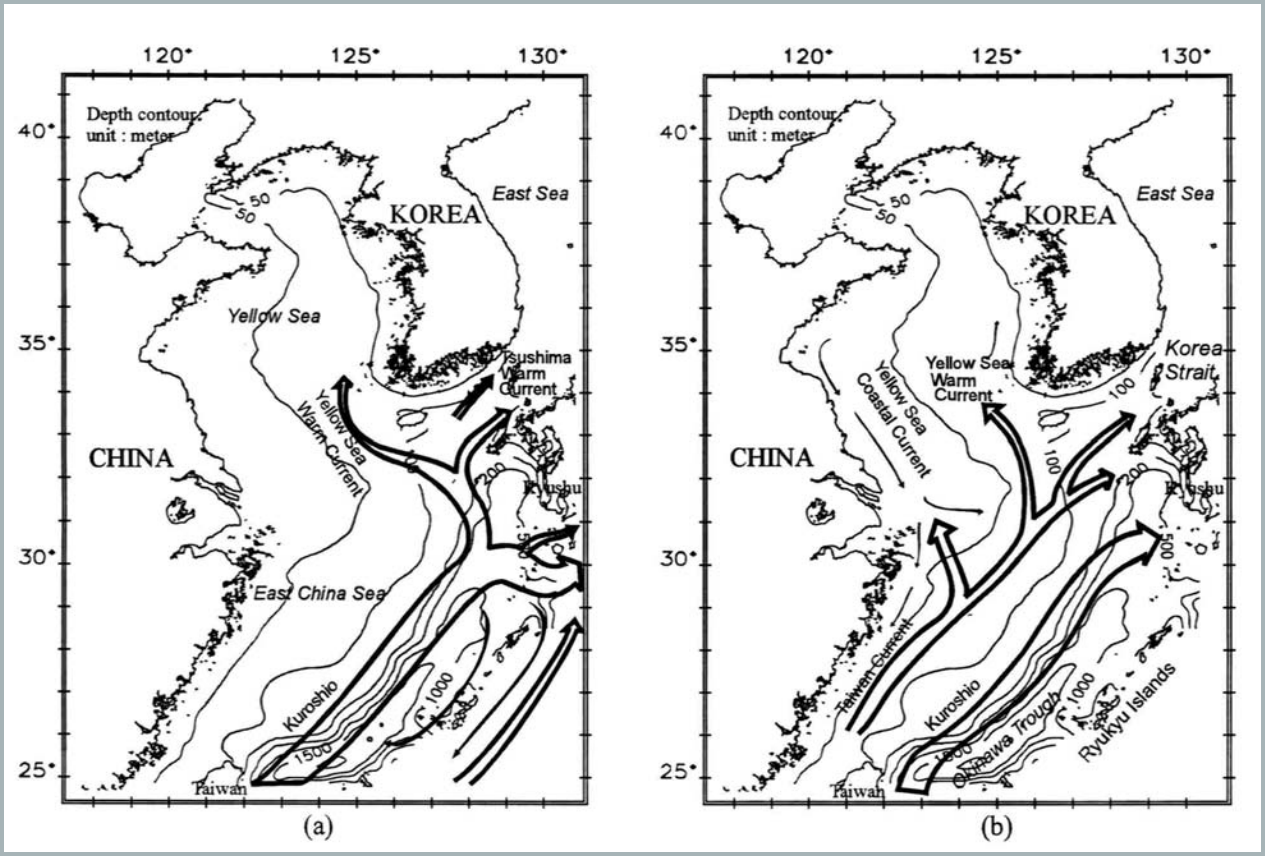
CAOCS Components

- *Atmosphere: MM5-V3.4*
- *Ocean: POM*
- *Land Surface: BATS*

CAOCS for East Asian Marginal Sea Prediction

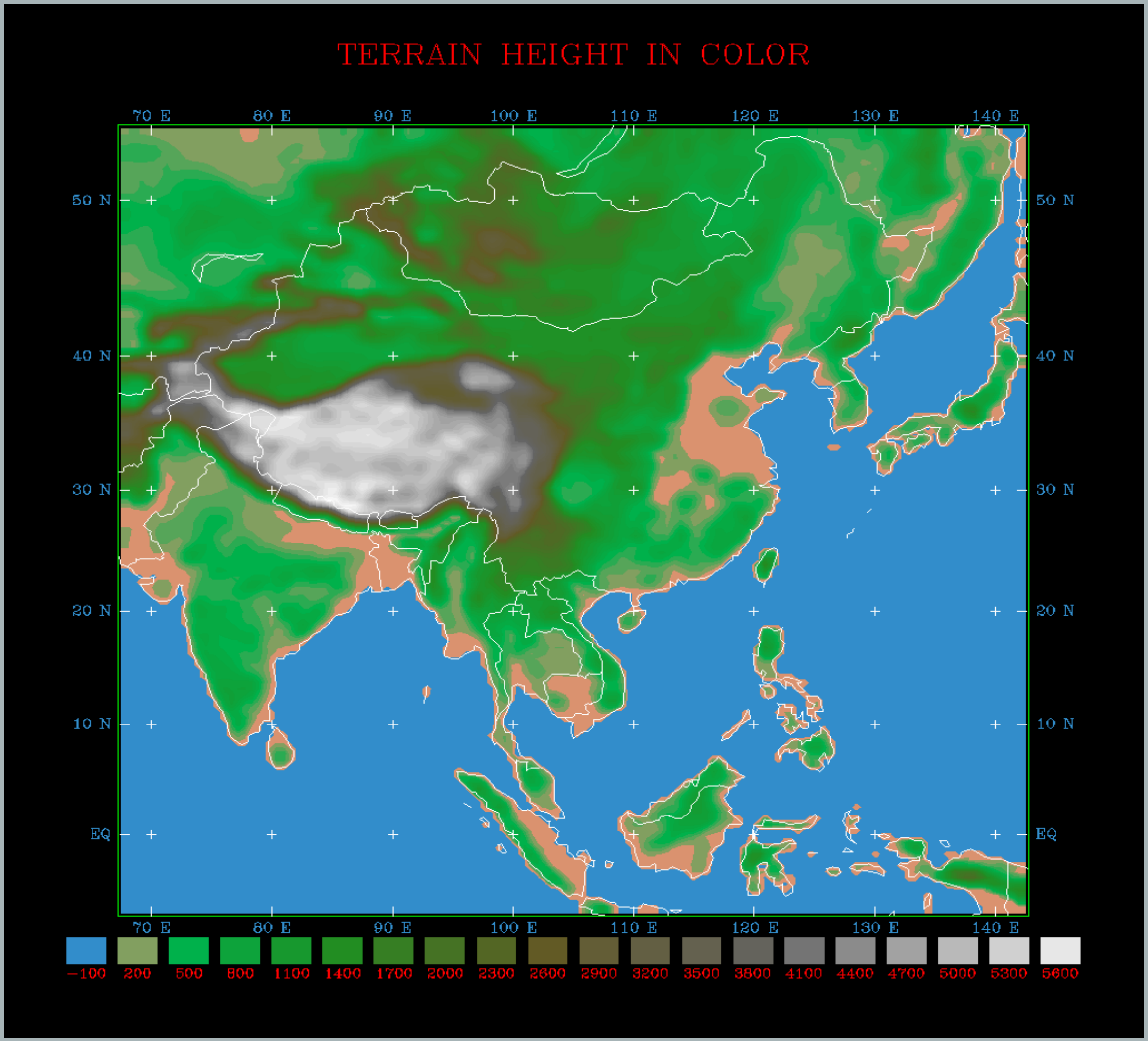
Chu et al. (1999, 2000)

East Asian Circulation System

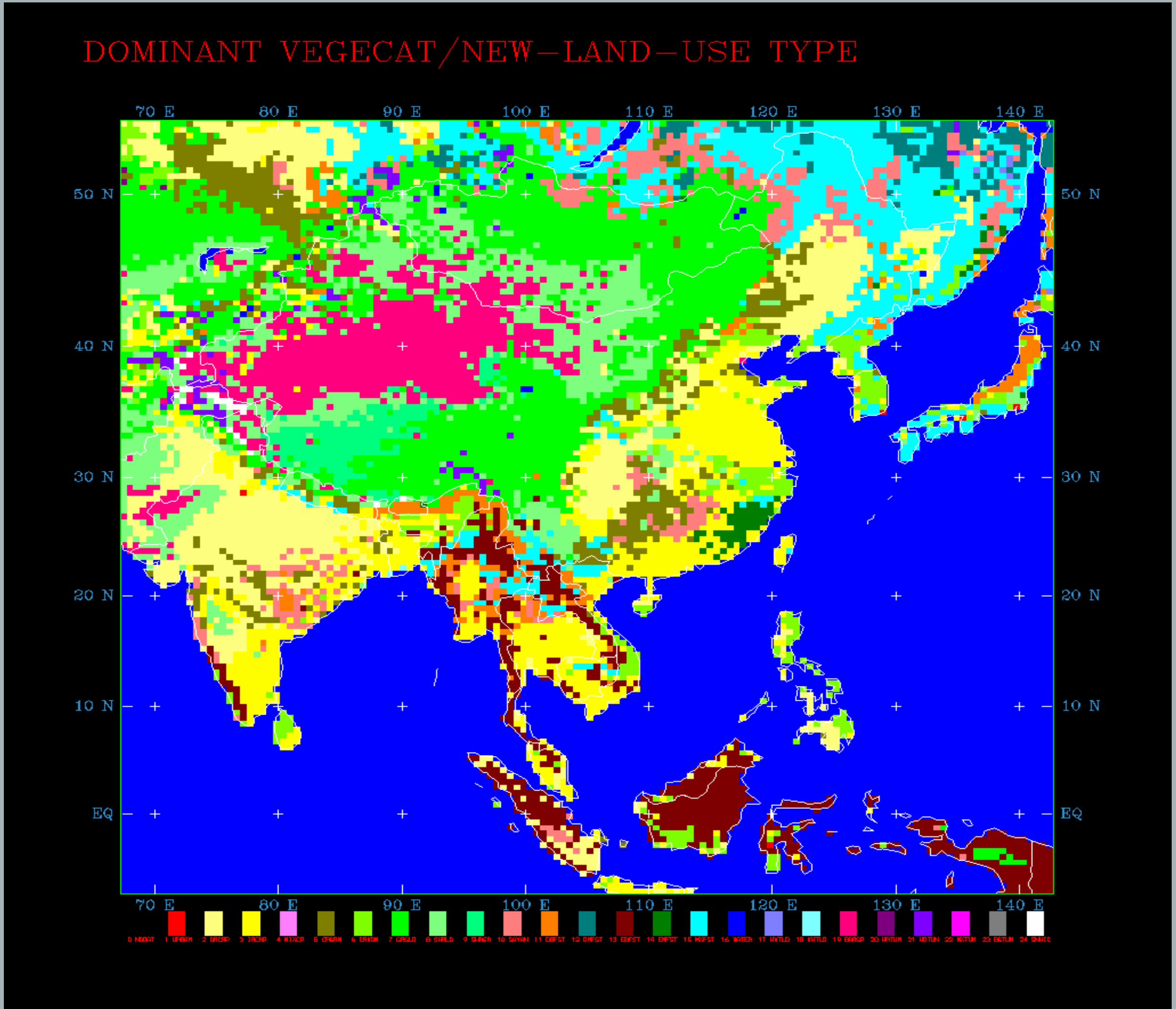


Nitani (1972) *Beardsley et al. (1983)*

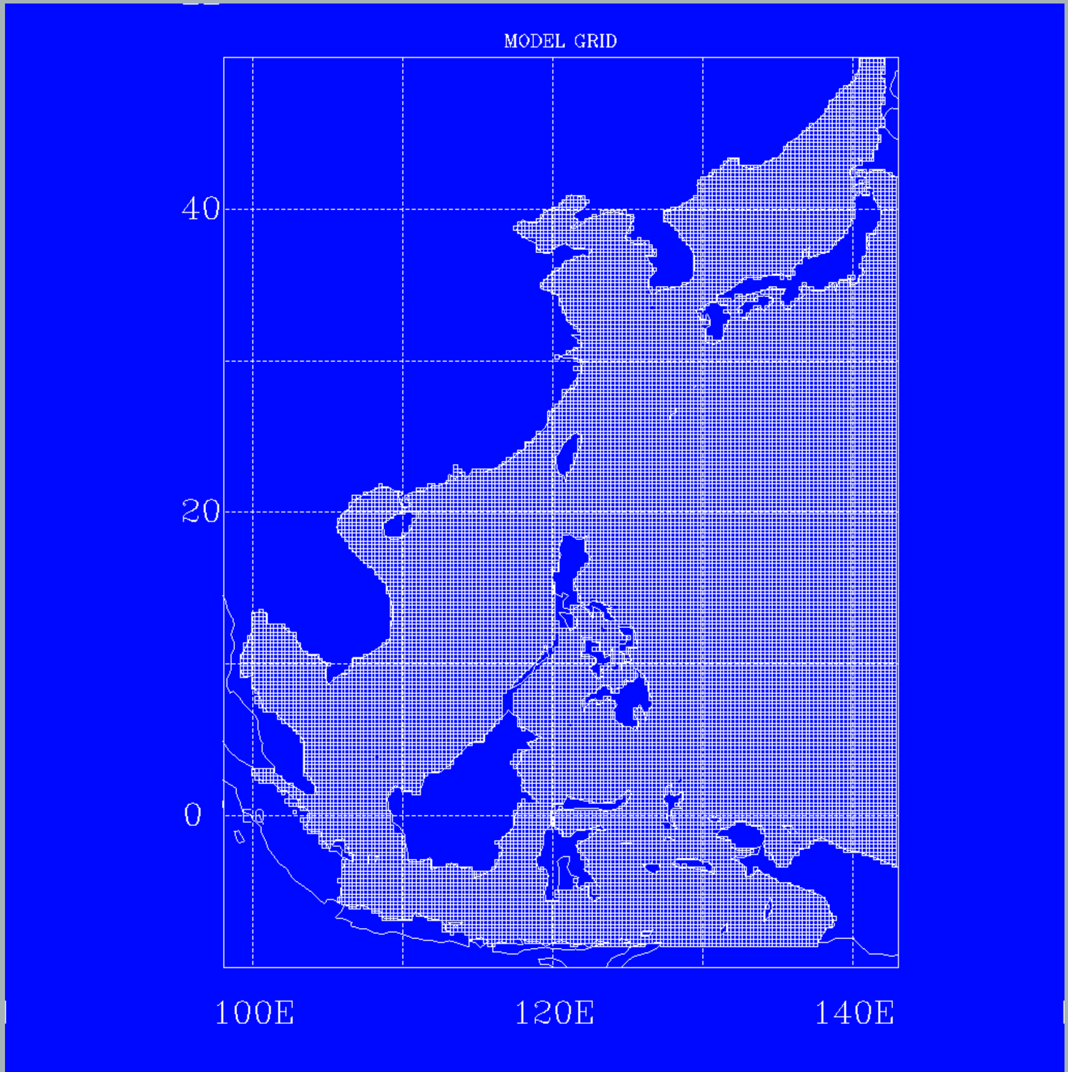
Area for Atmospheric Model



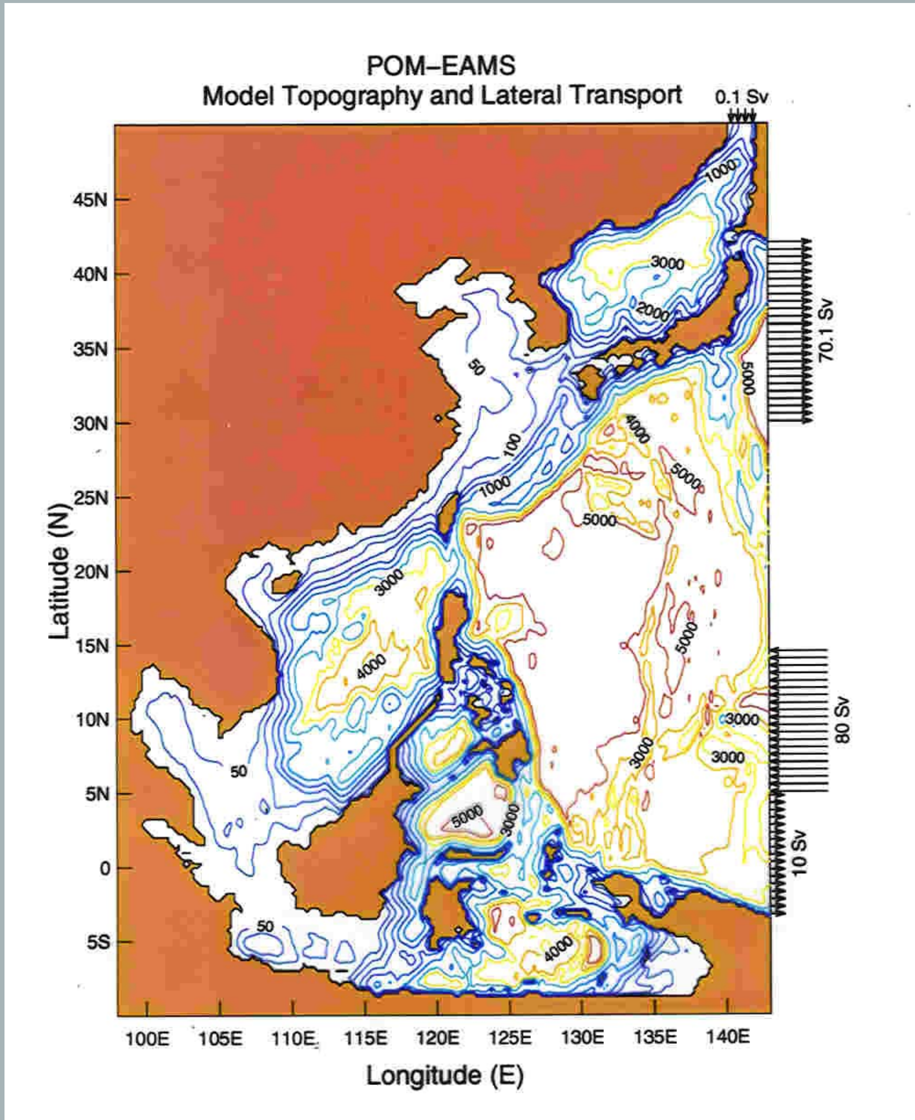
Distribution of Vegetation



Area for Ocean Model



Ocean Bottom



CAOCS Numerics

- *MM5V3.4*
  - *Resolution*
    - *Horizontal: 30 km*
    - *Vertical: 16 Pressure Levels*
  - *Time step: 2 min*
- *POM*
  - *Resolution*
    - *Horizontal: $1/6^o \times 1/6^o$*
    - *Vertical: 23 σ levels*
  - *Time Steps: 25 s, 15 min*

Ocean-Atmospheric Coupling

- *Surface fluxes (excluding solar radiation) are of opposite signs and applied synchronously to MM5 and POM*
- *MM5 and POM Update fluxes every 15 min*
- *SST for MM5 is obtained from POM*
- *Ocean wave effects (ongoing)*

Lateral Boundary Conditions

- *MM5: ECMWF T42*
- *POM: Lateral Transport at 142^{o}E*

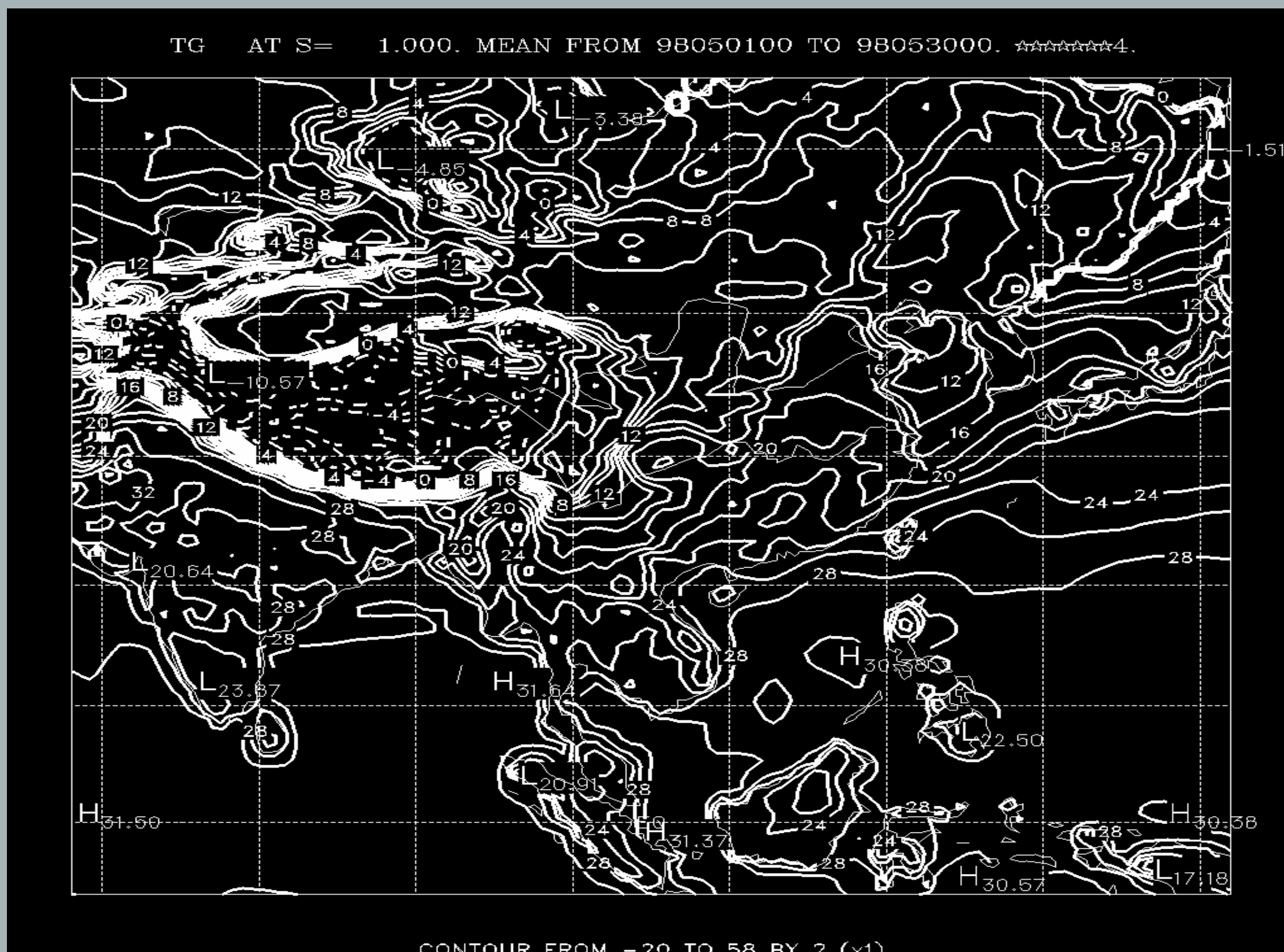
MM5 Initialization

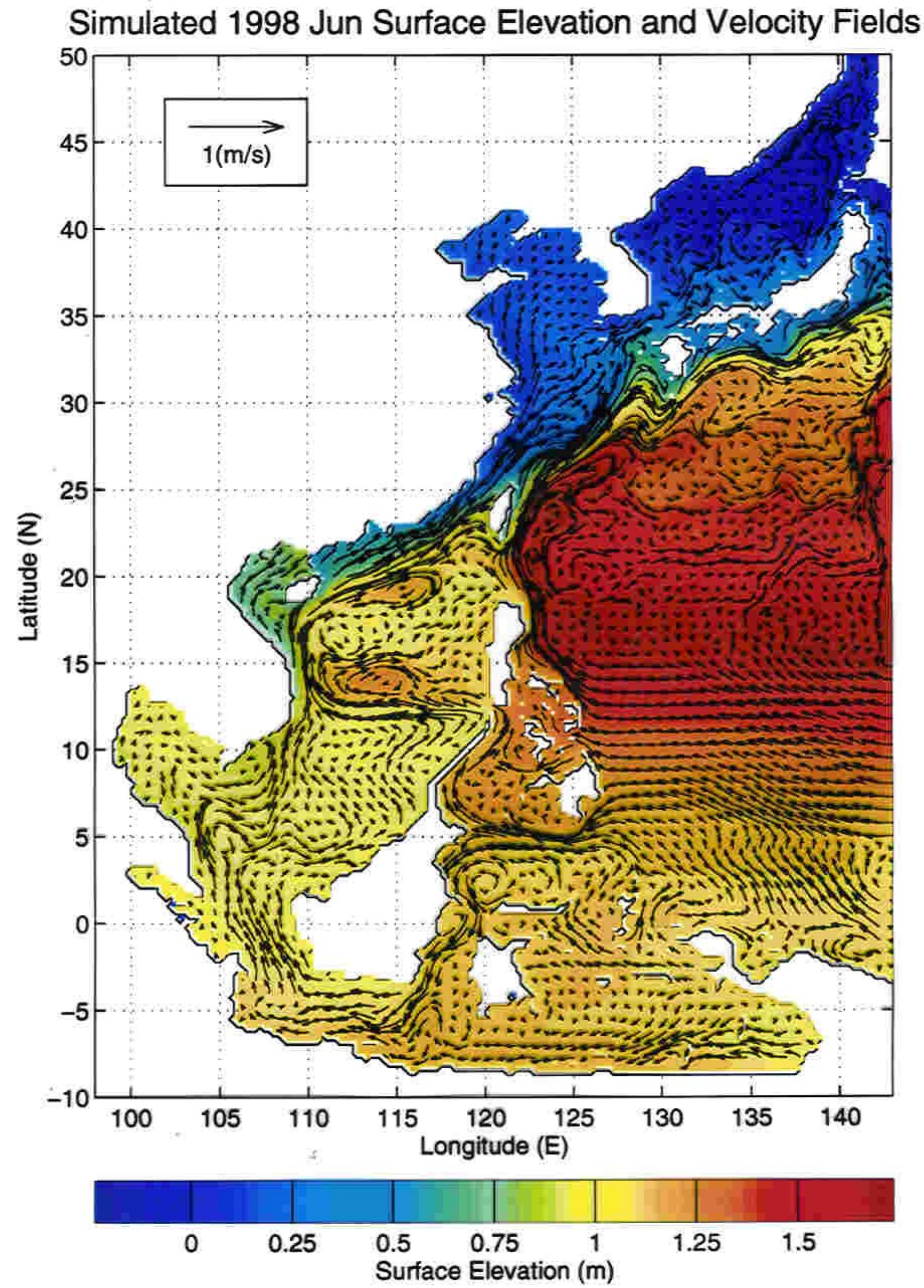
- *Initialized from: 30 April 1998 (ECMWF T42)*

Three-Step Initialization of POM

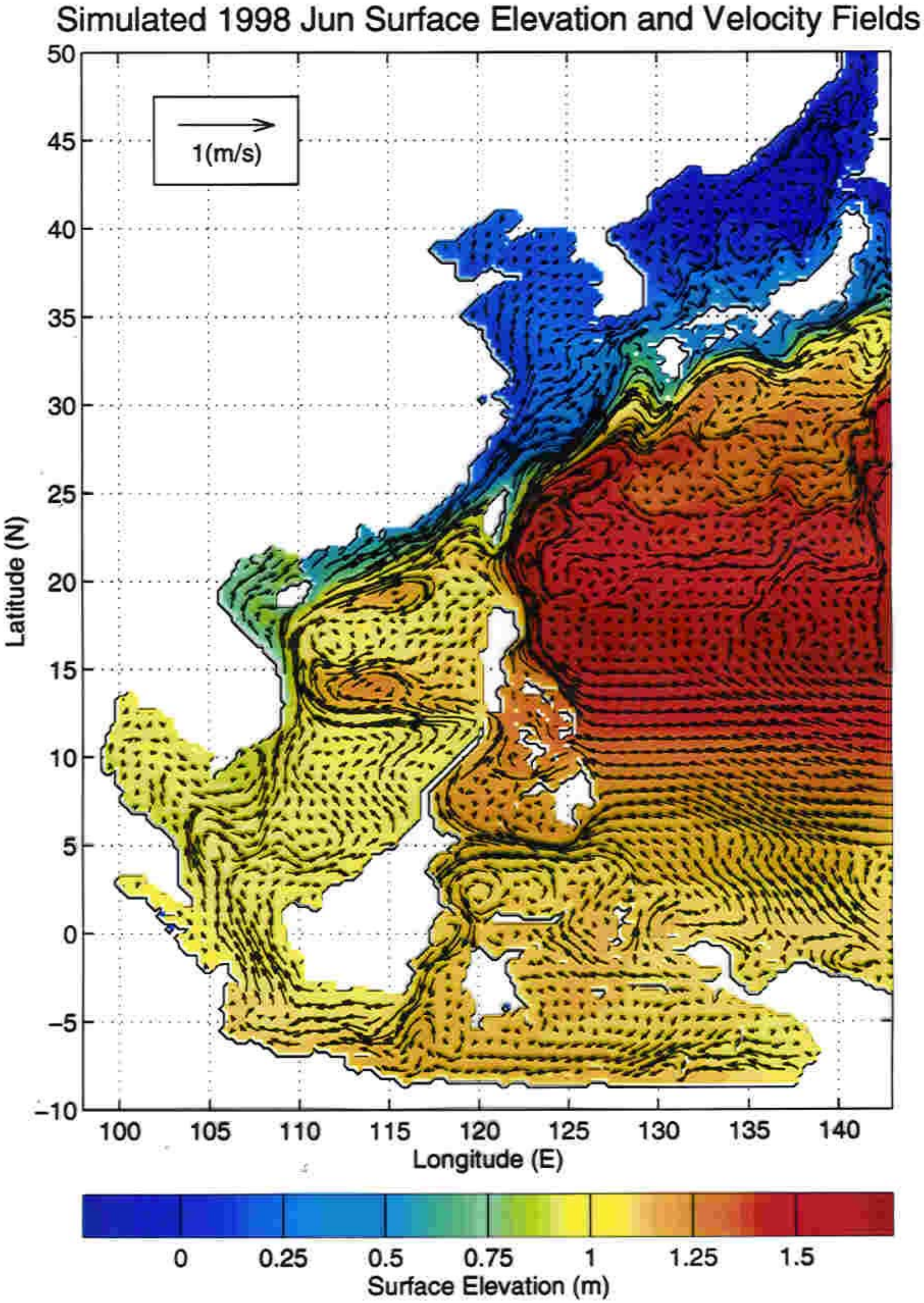
- *(1) Spin-up*
  - *Initial conditions: annual mean (T,S) + zero velocity*
  - *Climatological annual mean winds + Restoring type thermohaline flux (2 years)*
- *(2) Climatological Forcing*
  - *Monthly mean winds + thermohaline fluxes from COADS (3 years)*
- *(3) Synoptic Forcing*
  - *Winds and thermohaline fluxes from NCEP (1/1/96 – 4/30/98)*
- *(4) The final state of the previous step is the initial state of the following step*

Simulated Surface Air Temperature, May 98



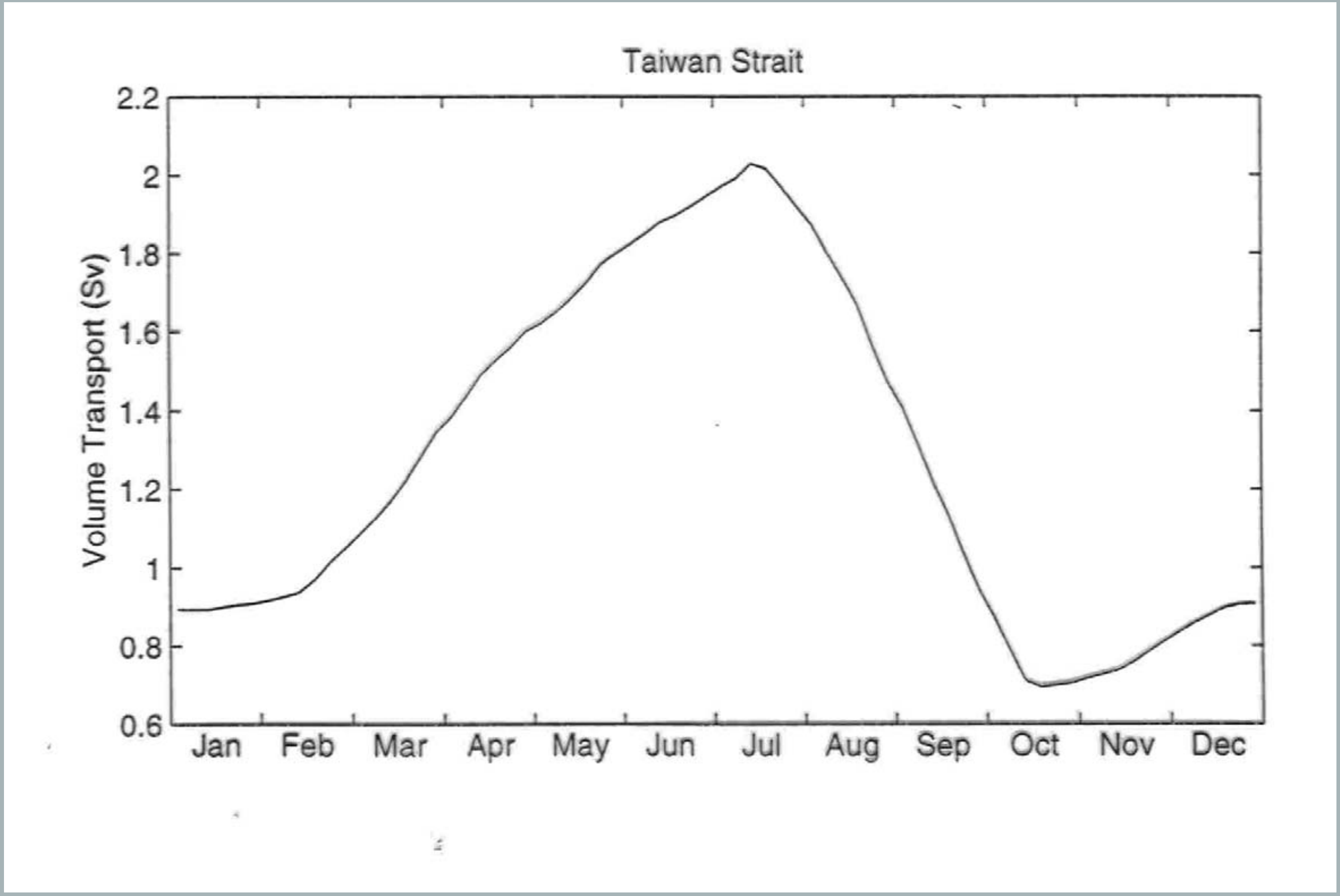


Simulated 1998 Jun Surface Elevation and Velocity Fields
1(m/s)
Latitude (N)
Longitude (E)
Surface Elevation (m)

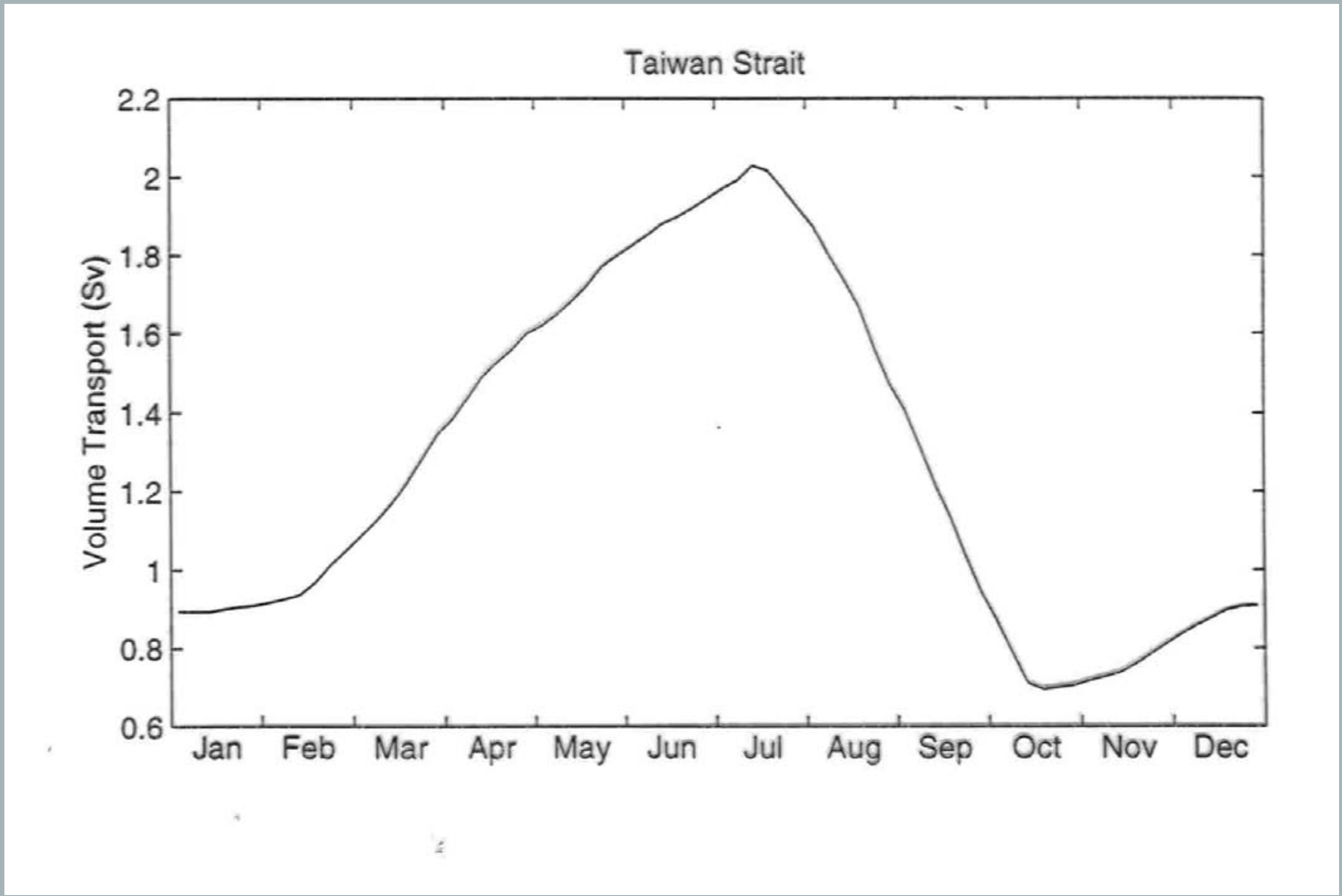


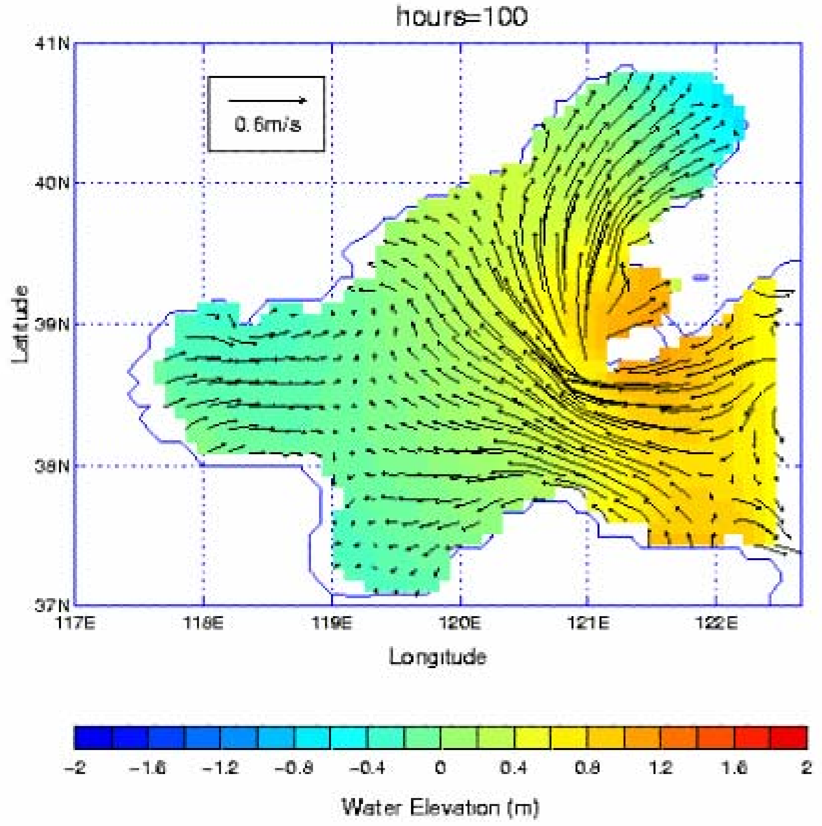
Simulated 1998 Jun Surface Elevation and Velocity Fields
1(m/s)
Latitude (N)
Longitude (E)
Surface Elevation (m)
0
0.25
0.5
0.75
1
1.25
1.5

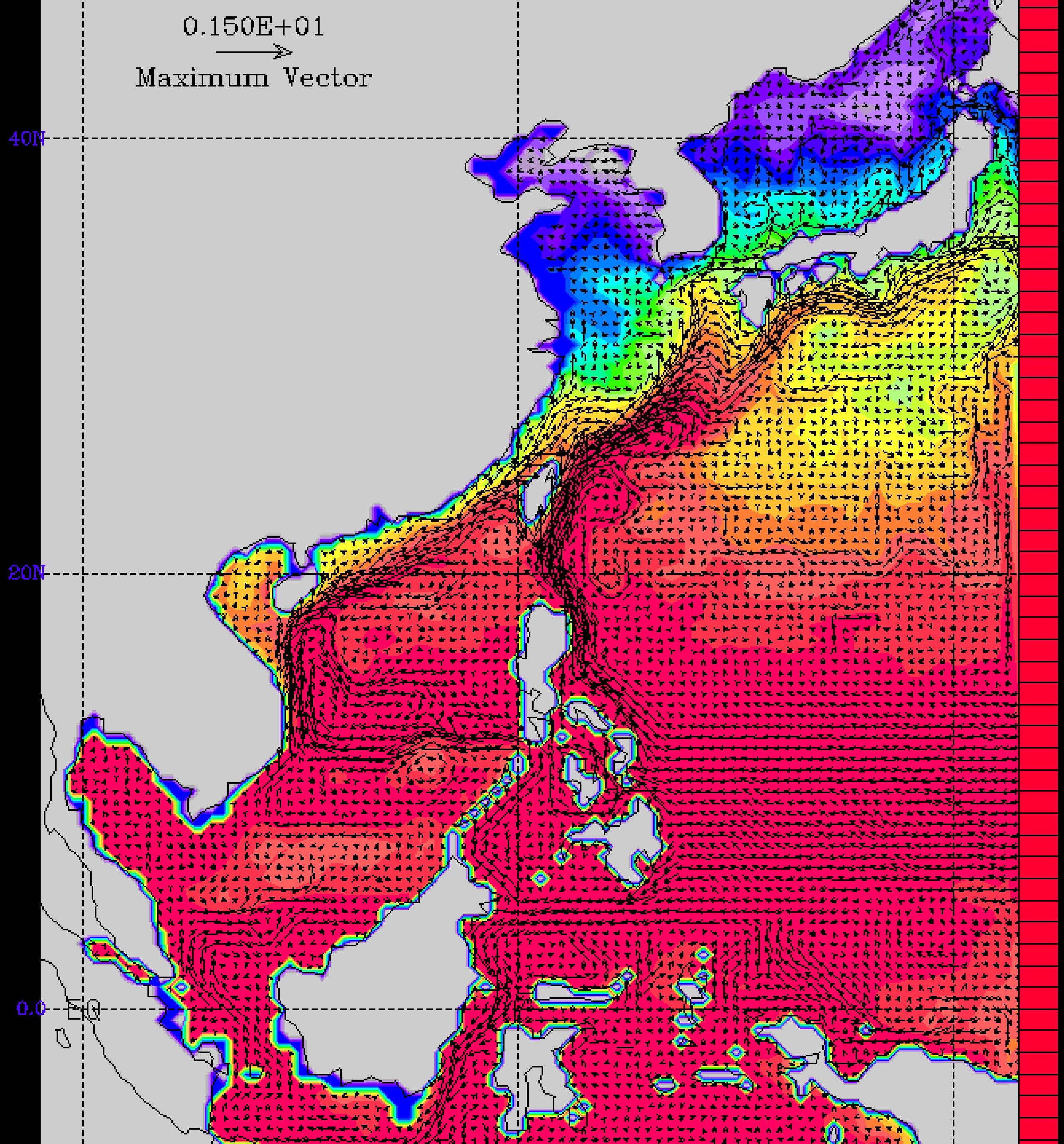
Volume Transport (Sv) Through Taiwan Strait

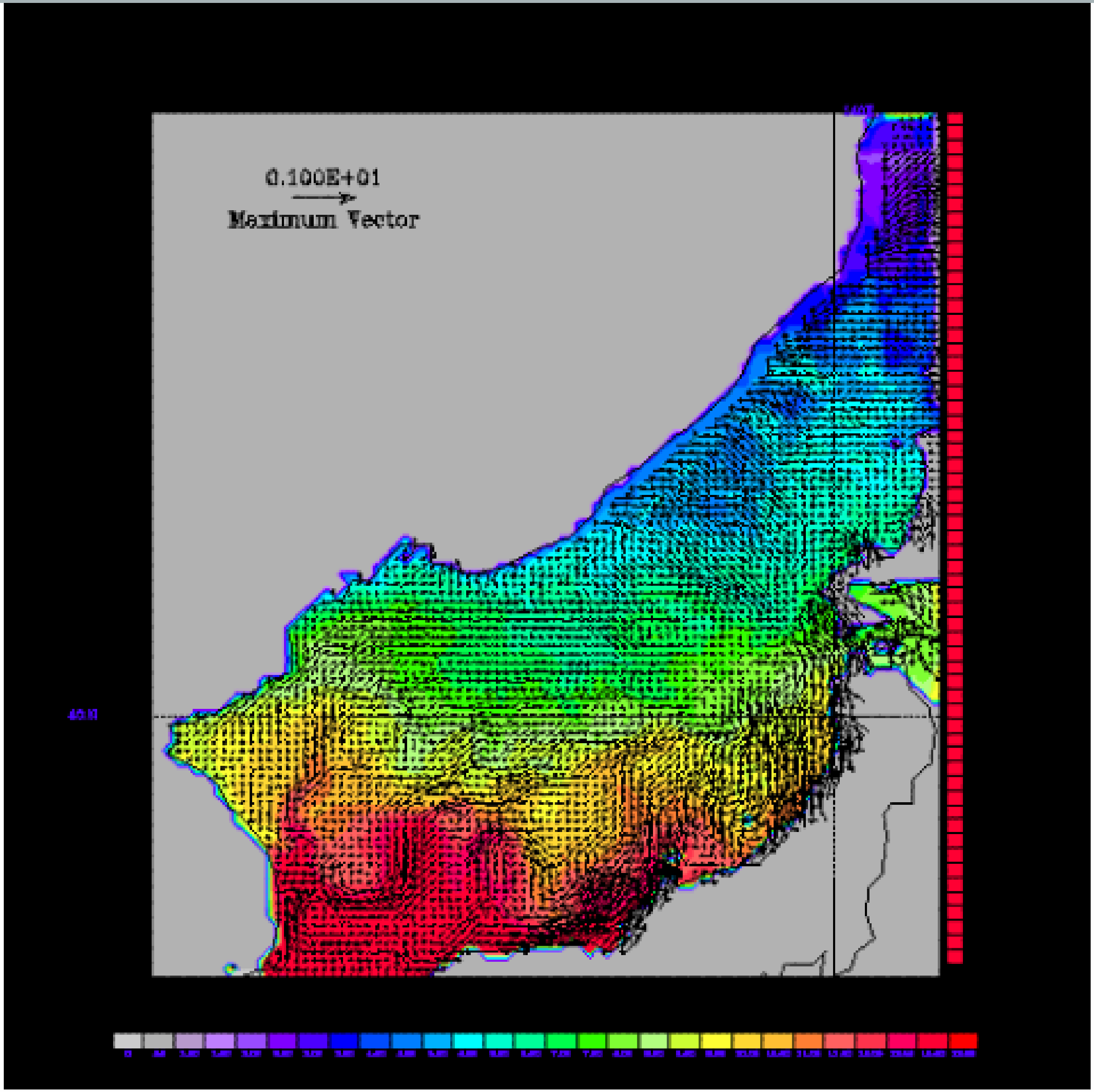


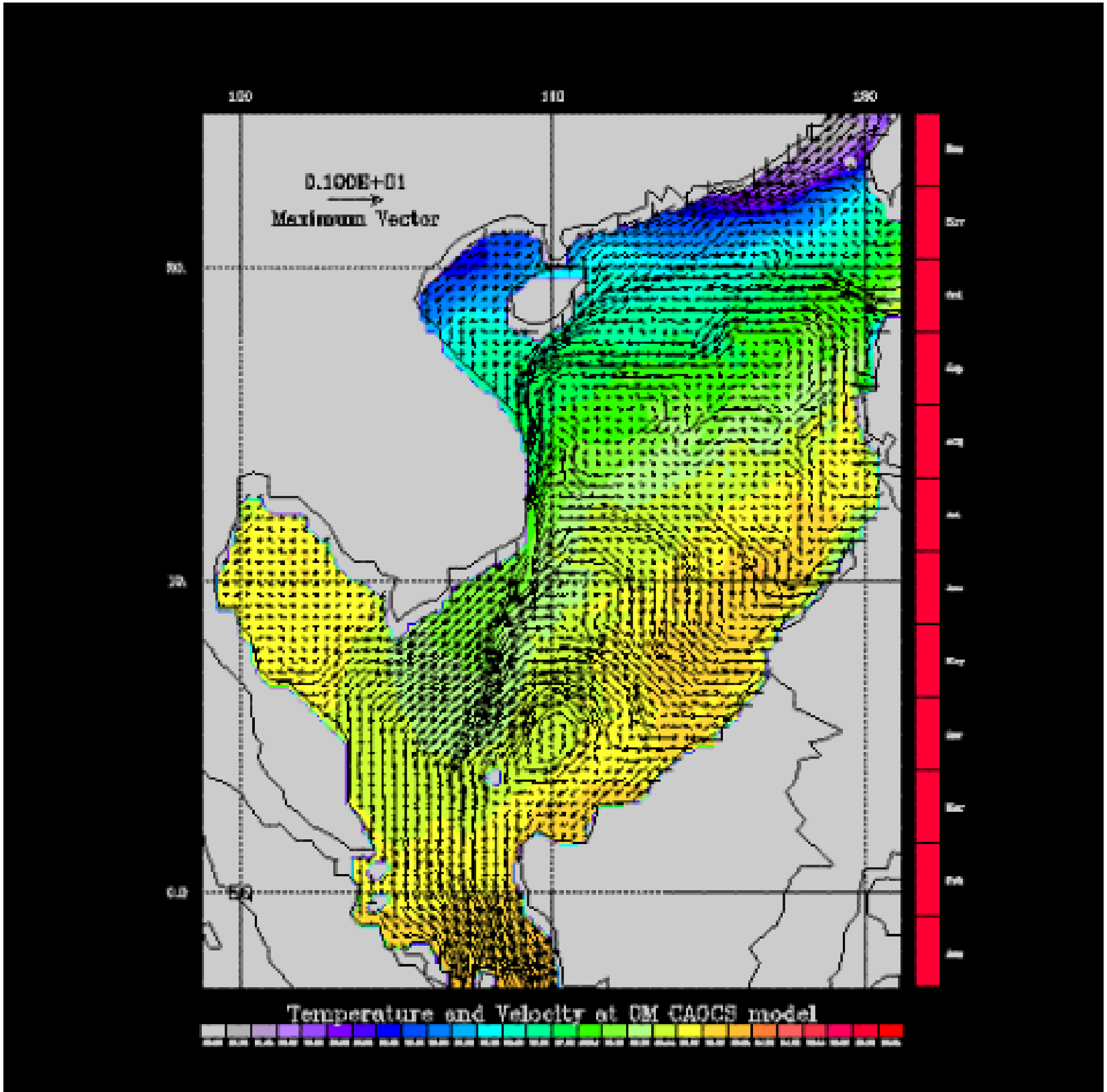
Volume Transport (Sv) Through Korean/Tsushima Strait




hours=100
0.6m/s
41N
40N
39N
38N
37N
117E
118E
119E
120E
121E
122E
Latitude
Longitude
-2
-1.6
-1.2
-0.8
-0.4
0
0.4
0.8
1.2
1.6
2
Water Elevation (m)

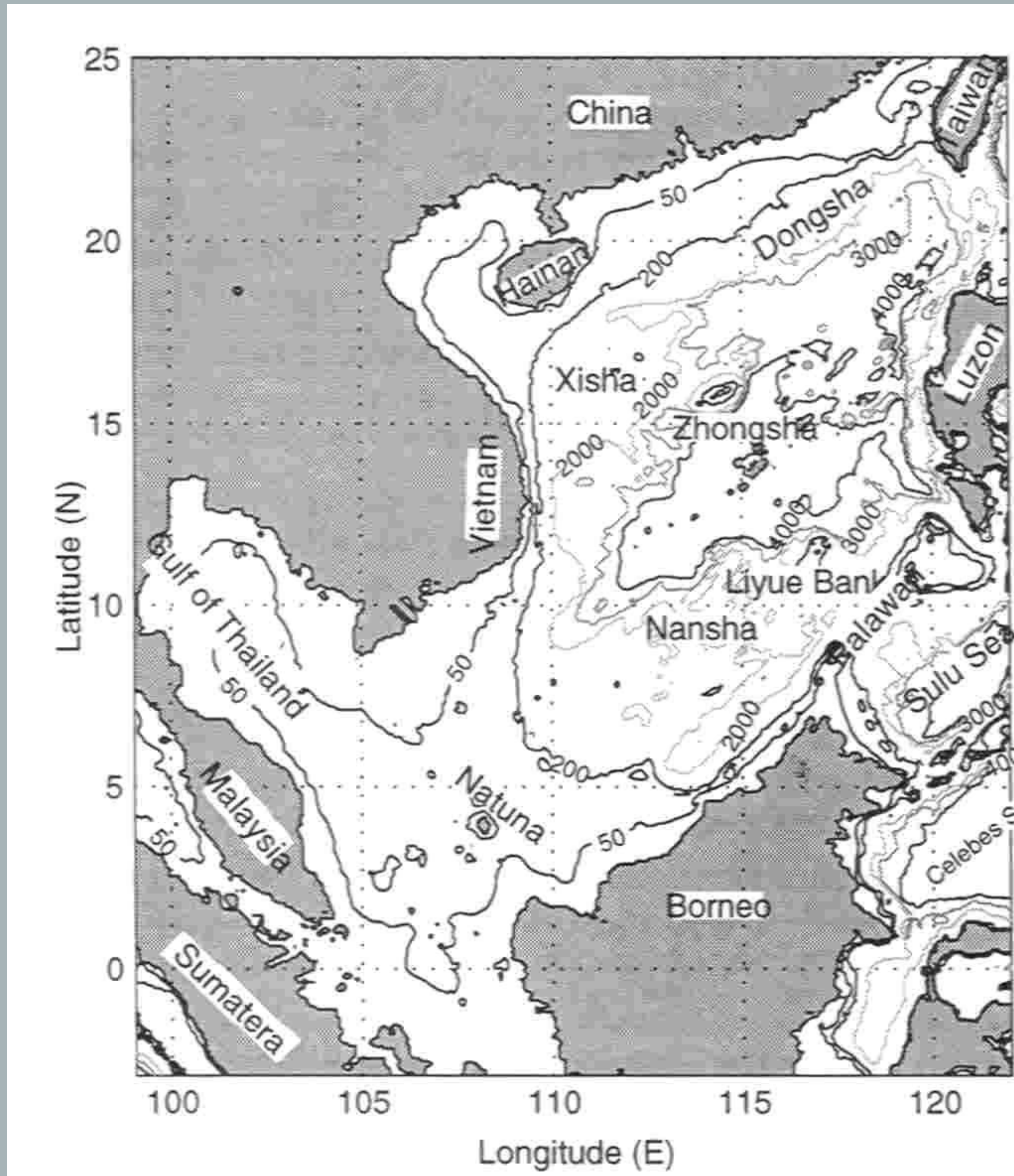

0.150E+01
Maximum Vector
40N
20N
0.0
EQ


0.100E+01
Maximum Vector


100
110
120
0.100E+01
Maximum Vector
Temperature and Velocity at OM CAOCS model

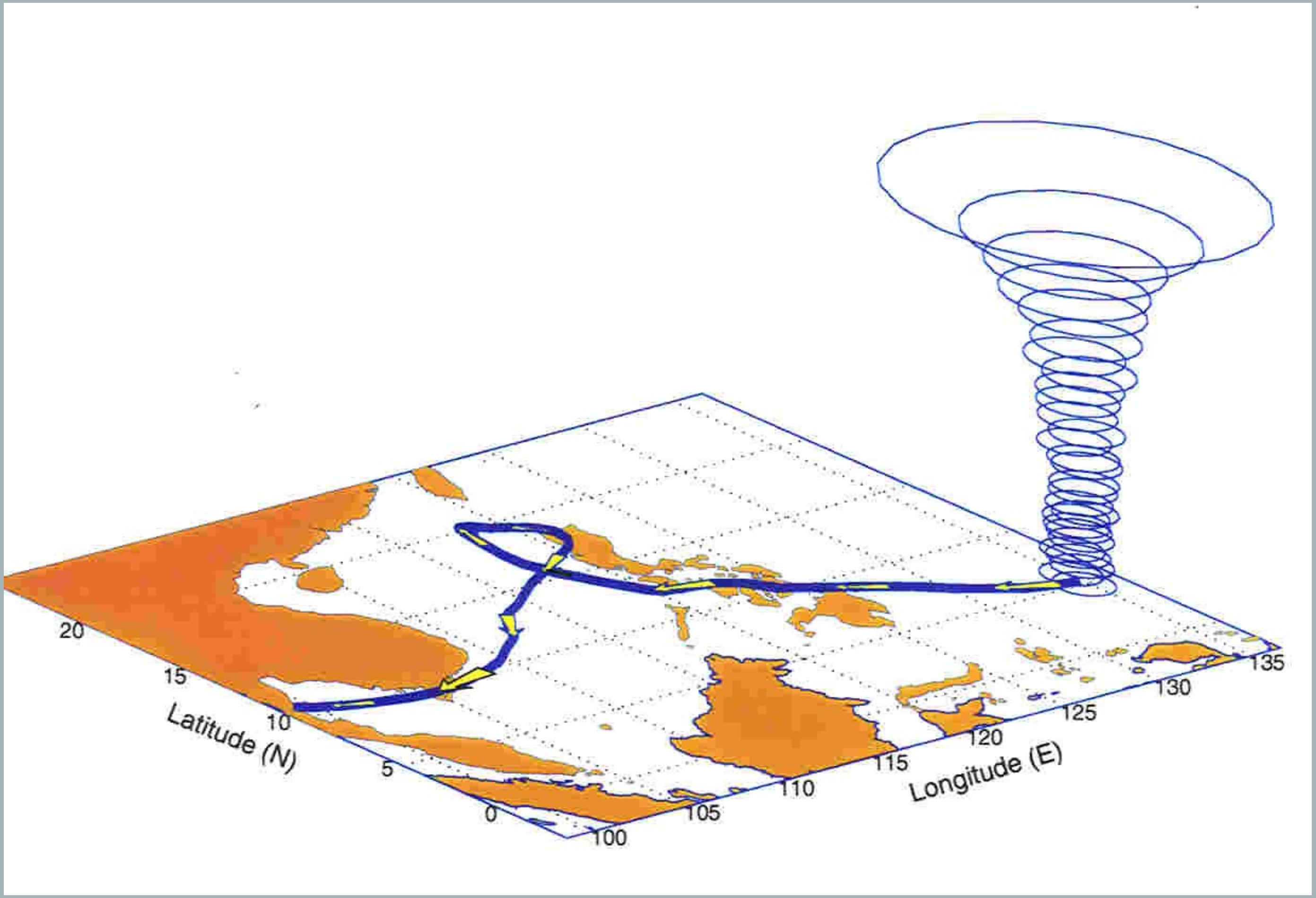
(6) Severe Weather Effect

- *Response of the South China Sea to tropical cyclone Ernie 1996*
- *Chu et al. 2000 (JGR)*



China
Hainan
Dongsha
Xisha
Zhongsha
Vietnam
Luzon
Gulf of Thailand
Liyue Bank
Nansha
Sulu Sea
Malaysia
Natuna
Borneo
Sumatera
Celebes Sea
50
200
2000
3000
4000
Latitude (N)
Longitude (E)
25
20
15
10
5
0
100
105
110
115
120

Tropical Cyclone Ernie 1996



Tropical Cyclone Wind Profile Model (Carr & Elsberry 1997, MWR)

$$v_c(r) = \frac{f_0}{2}\left[R_0\left(\frac{R_0}{r}\right)^X - r\right]\frac{a^4}{1+a^4},$$

$$u_c(r) = \tan(\gamma)v_c(r),$$

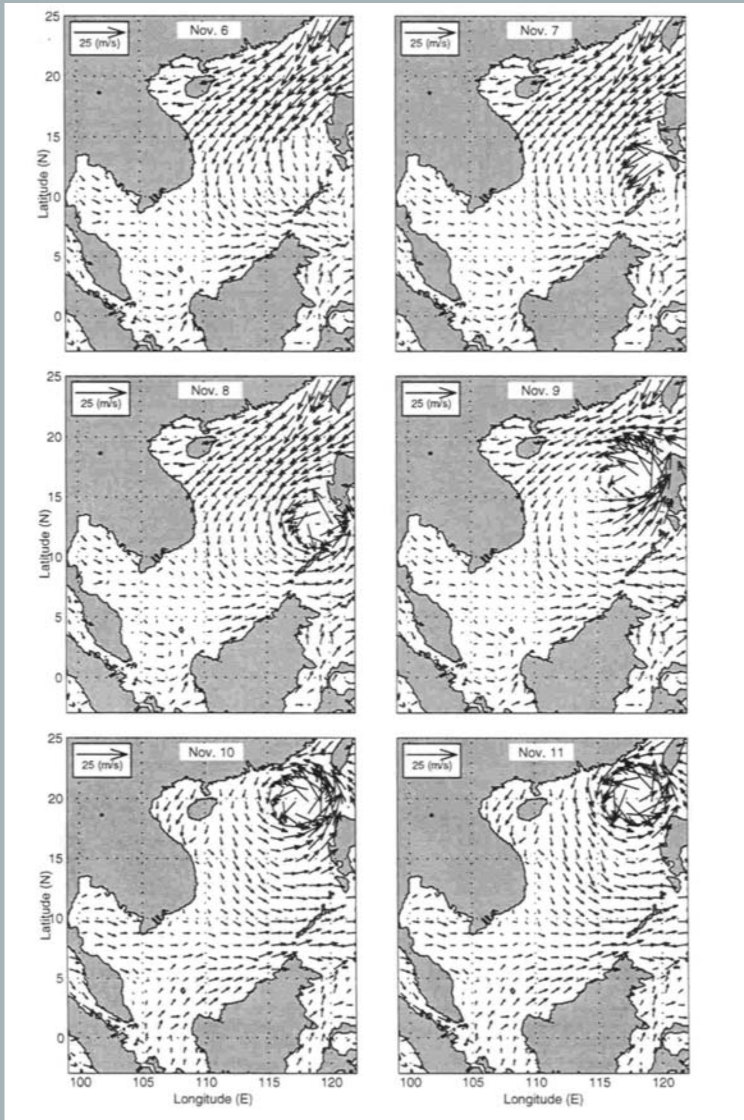
- *r ~ horizontal distance to the storm center*
- *(u_c, v_c) ~ radial and tangential velocities*
- *γ ~ wind inflow angle to storm center*
- *$a = r/R_m$, scaling factor*
- *X ~ parameter*

Surface Wind Field

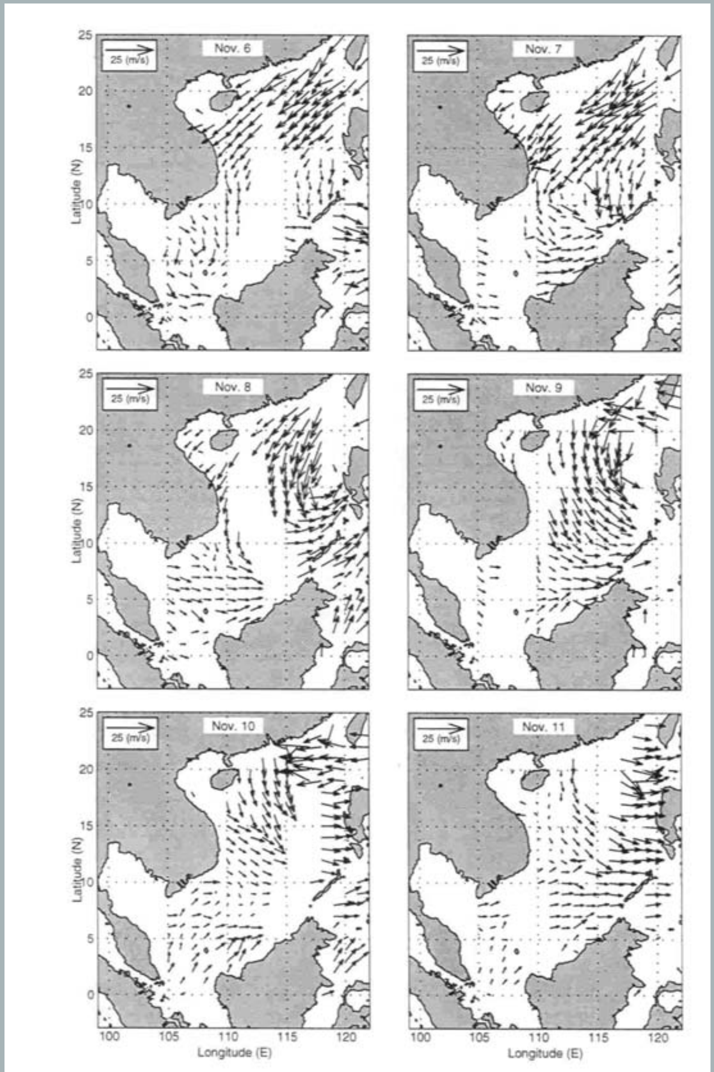
$$\mathbf{V} = (1-\varepsilon)(\mathbf{V}_c + \mathbf{V}_t) + \varepsilon \mathbf{V}_{bg},$$

- $\mathbf{V}_c$ ~ Wind field relative to the storm center (from the wind profile model)
- $\mathbf{V}_t$ ~ Strom translation velocity
- $\mathbf{V}_{bg}$ ~ Back ground wind field

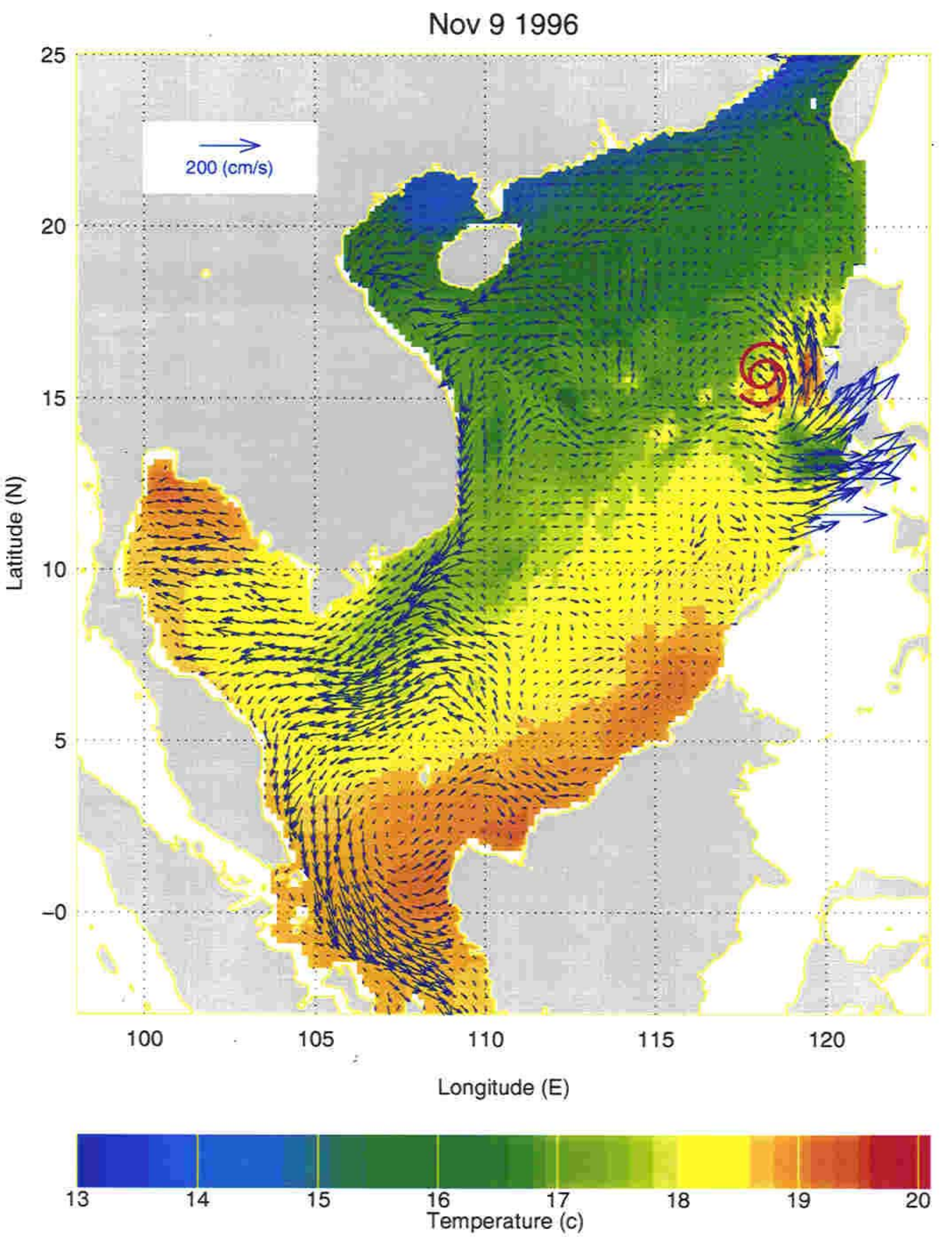
Computed Wind Field

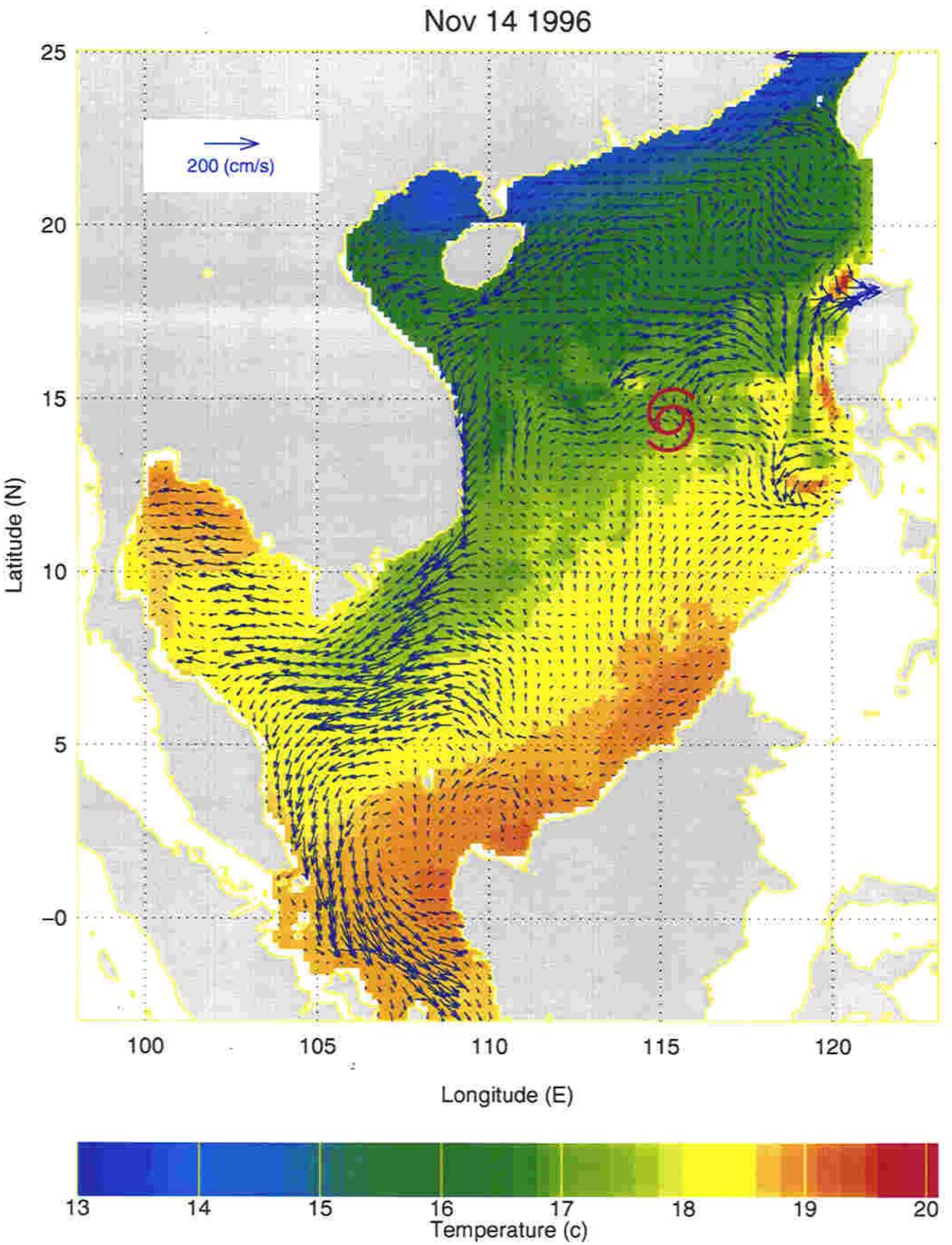


NSCAT Winds

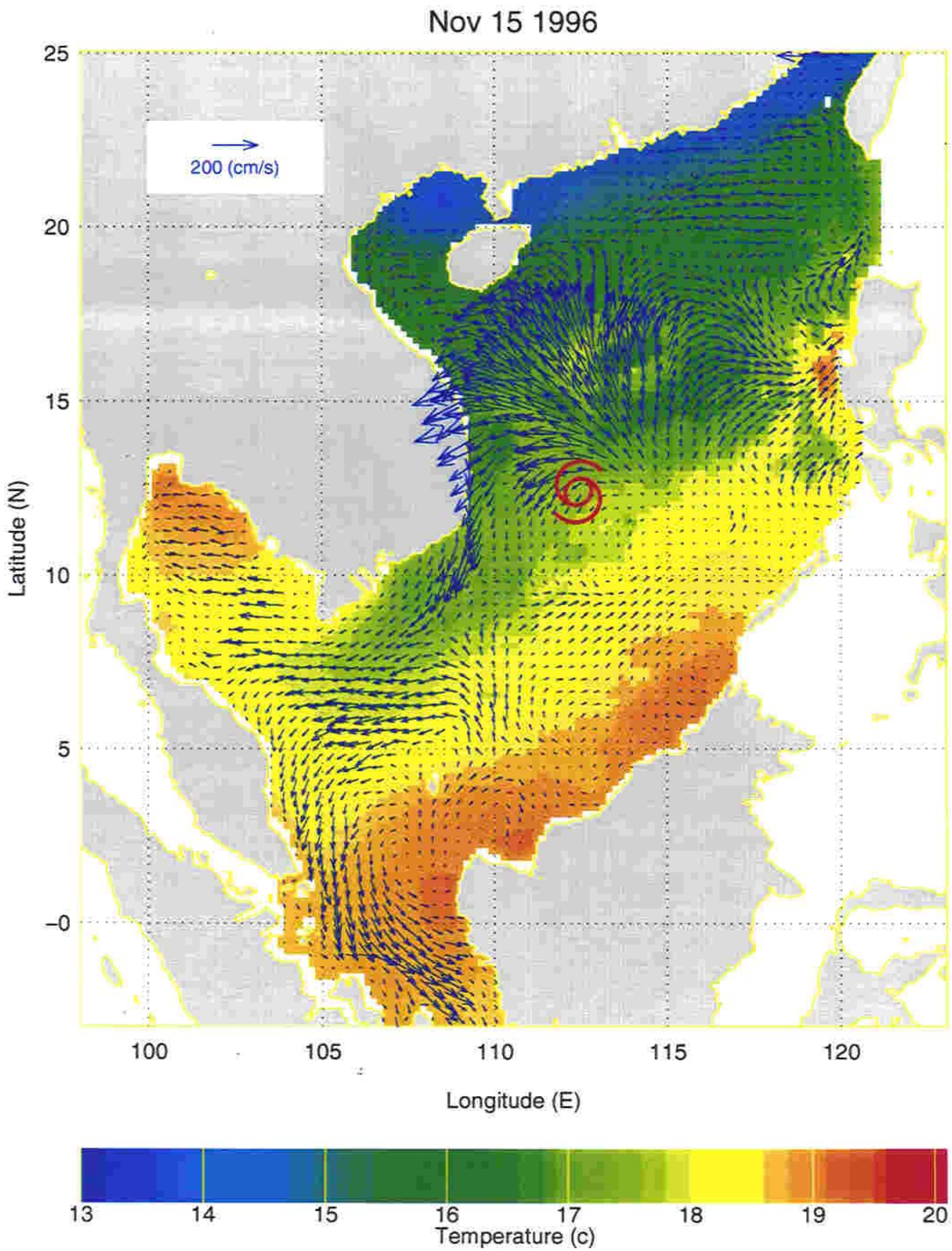


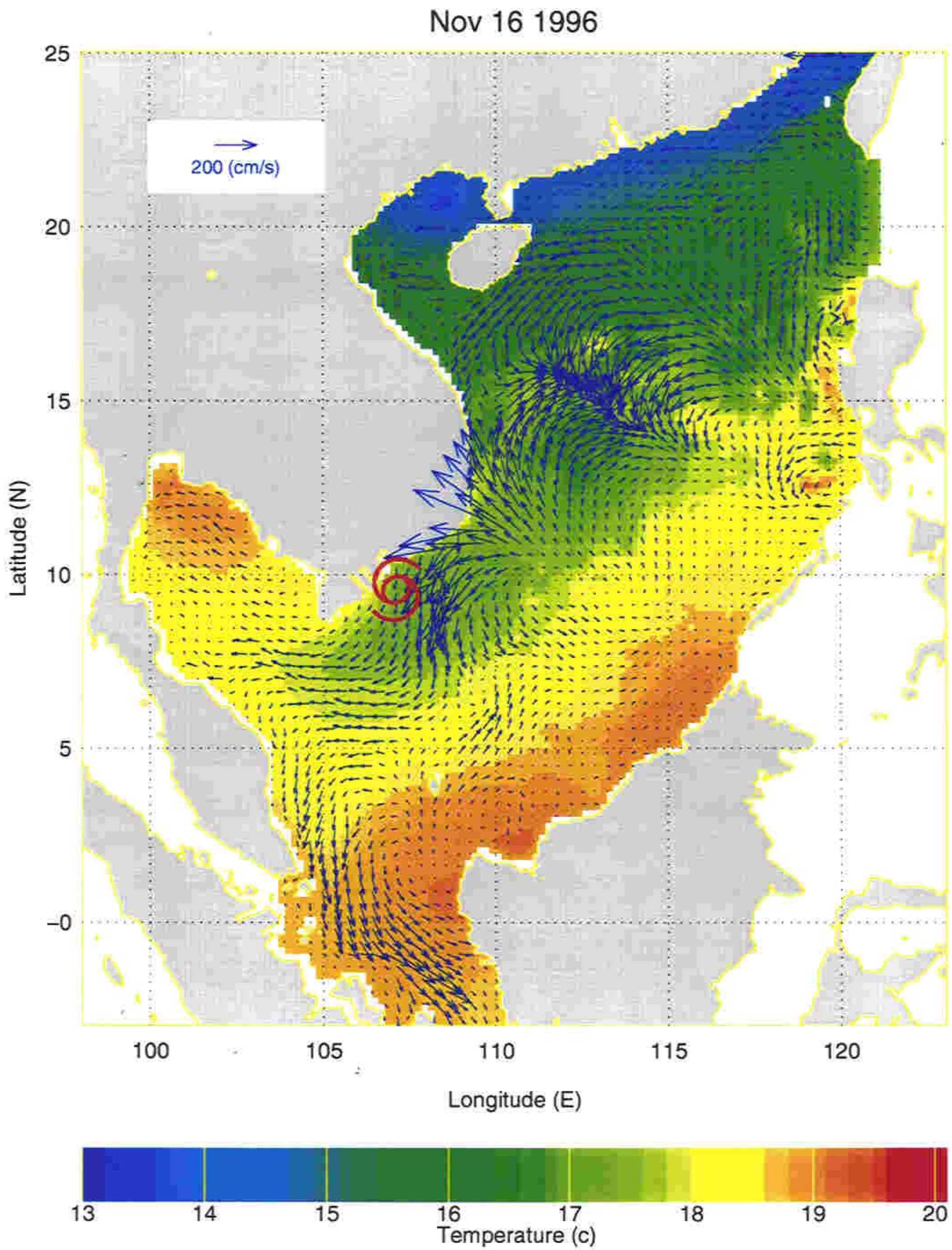
25 (m/s)
Nov. 6
Nov. 7
Nov. 8
Nov. 9
Nov. 10
Nov. 11
Latitude (N)
Longitude (E)
25
20
15
10
5
0
100
105
110
115
120


Nov 9 1996
200 (cm/s)
Latitude (N)
25
20
15
10
5
−0
100
105
110
115
120
Longitude (E)
13
14
15
16
17
18
19
20
Temperature (c)



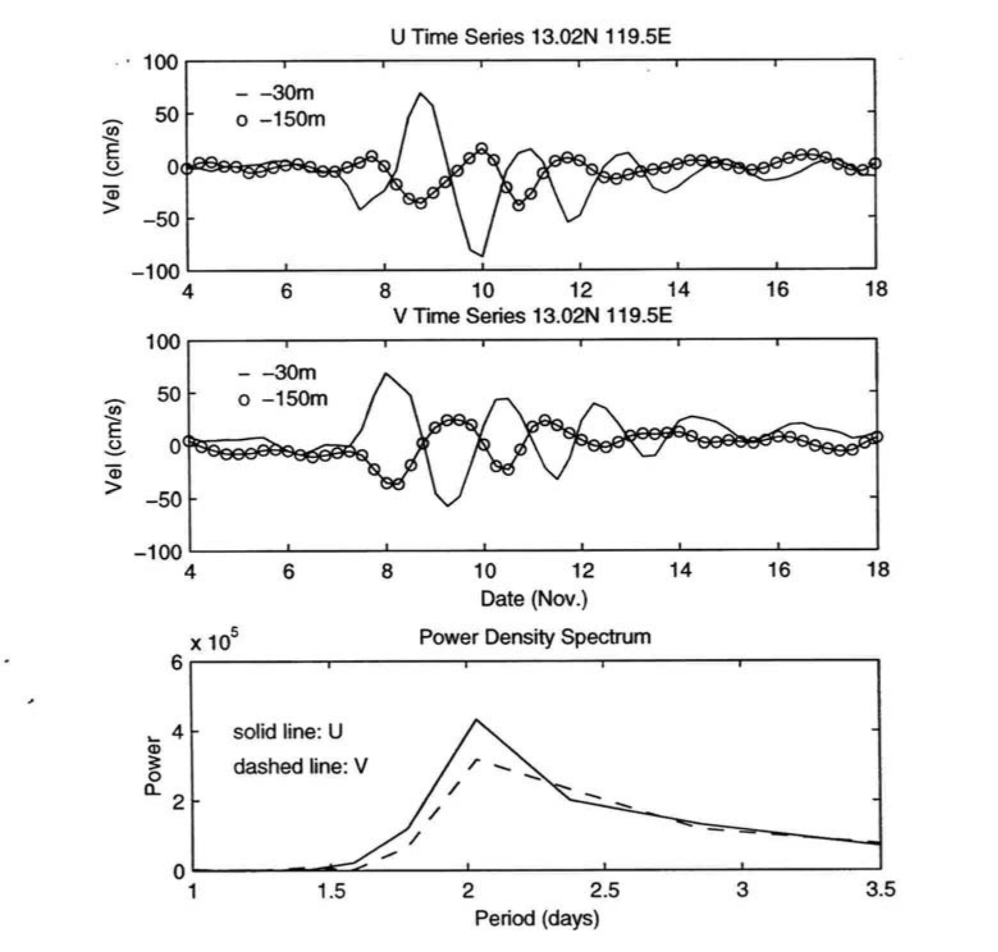
Nov 14 1996
200 (cm/s)
25
20
15
10
5
−0
Latitude (N)
100
105
110
115
120
Longitude (E)
13
14
15
16
17
18
19
20
Temperature (c)

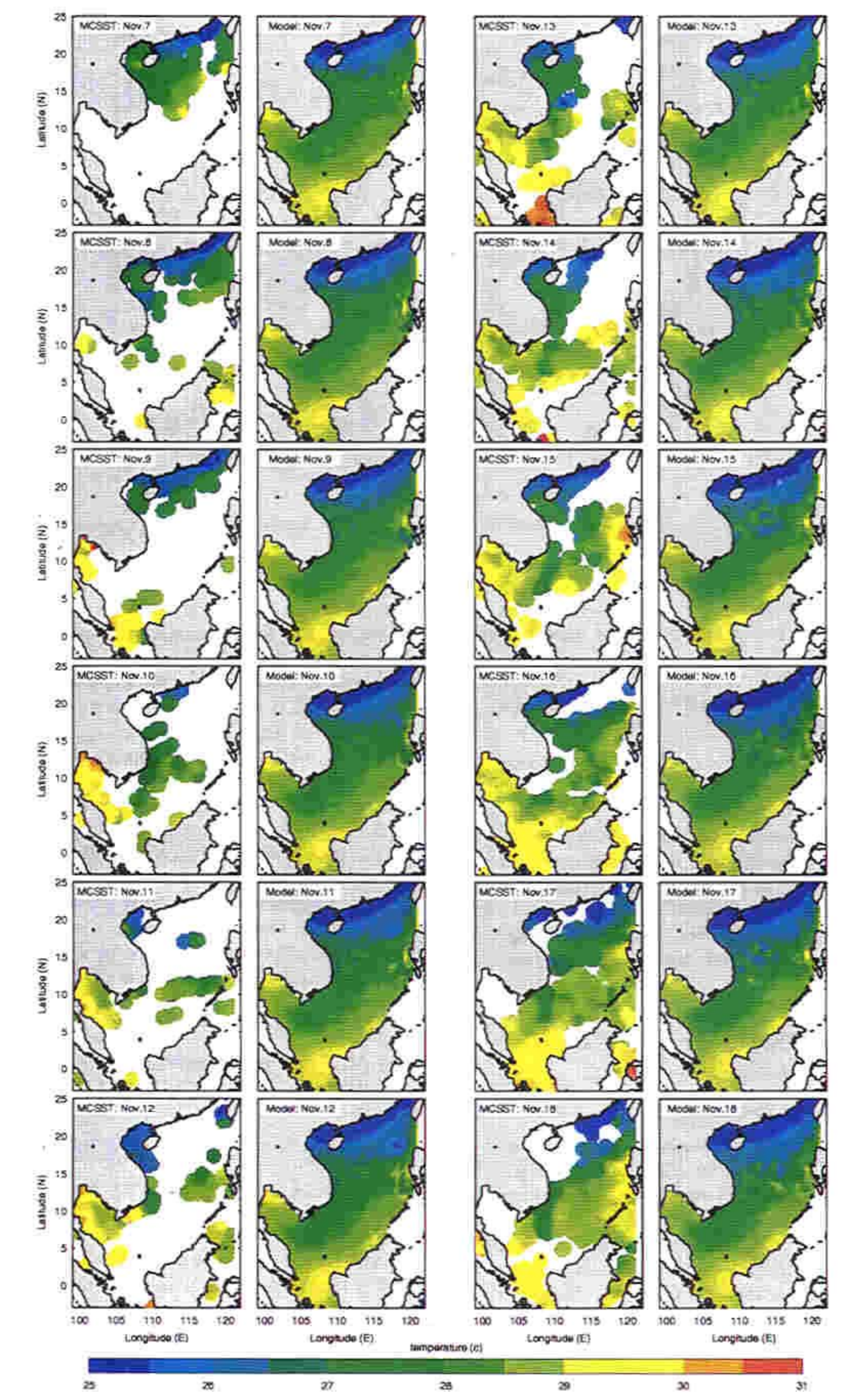

Nov 15 1996
200 (cm/s)
25
20
15
10
5
-0
Latitude (N)
100
105
110
115
120
Longitude (E)
13
14
15
16
17
18
19
20
Temperature (c)



Nov 16 1996
200 (cm/s)
Latitude (N)
25
20
15
10
5
–0
100
105
110
115
120
Longitude (E)
13
14
15
16
17
18
19
20
Temperature (c)

Time series of velocity and power density at 13°N and 119.5°E from November 2 to 18, 1996





MCSST: Nov.7
Model: Nov.7
MCSST: Nov.13
Model: Nov.13
MCSST: Nov.8
Model: Nov.8
MCSST: Nov.14
Model: Nov.14
MCSST: Nov.9
Model: Nov.9
MCSST: Nov.15
Model: Nov.15
MCSST: Nov.10
Model: Nov.10
MCSST: Nov.16
Model: Nov.16
MCSST: Nov.11
Model: Nov.11
MCSST: Nov.17
Model: Nov.17
MCSST: Nov.12
Model: Nov.12
MCSST: Nov.18
Model: Nov.18
Latitude (N)
25
20
15
10
5
0
100
105
110
115
120
Longitude (E)
temperature (c)
25
26
27
28
29
30
31

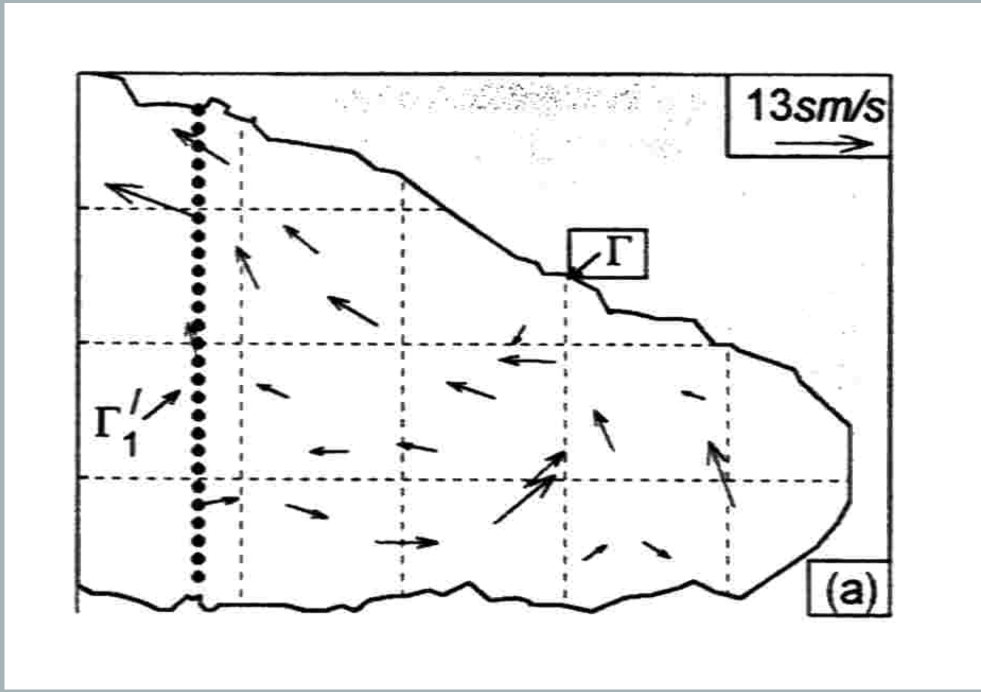
Comments

- *Pom has a capability to simulate response of coastal water to tropical cyclones.*

(7) Velocity Data Assimilation

Can we get the velocity signal from sparse and noisy data?

- *Black Sea*



Reconstruction of Velocity Field in Open Domain

Chu (2000)

Chu and Ivanov (2001a,b)

Flow Decomposition

- *2 D Flow (Helmholtz)*

$$\mathbf{u}_H = \mathbf{r} \times \boldsymbol{\nabla}_H A_1 + \boldsymbol{\nabla}_H A_3$$

- *3D Flow (Toroidal & Poloidal): Very popular in astrophysics*
- $$\mathbf{u} = \mathbf{r} \times \boldsymbol{\nabla} A_1 + \mathbf{r}\mathbf{A}_2 + \boldsymbol{\nabla} A_3$$

3D Incompressible Flow

-
- *When* $\nabla \bullet \mathbf{u} = 0$
- *We have*

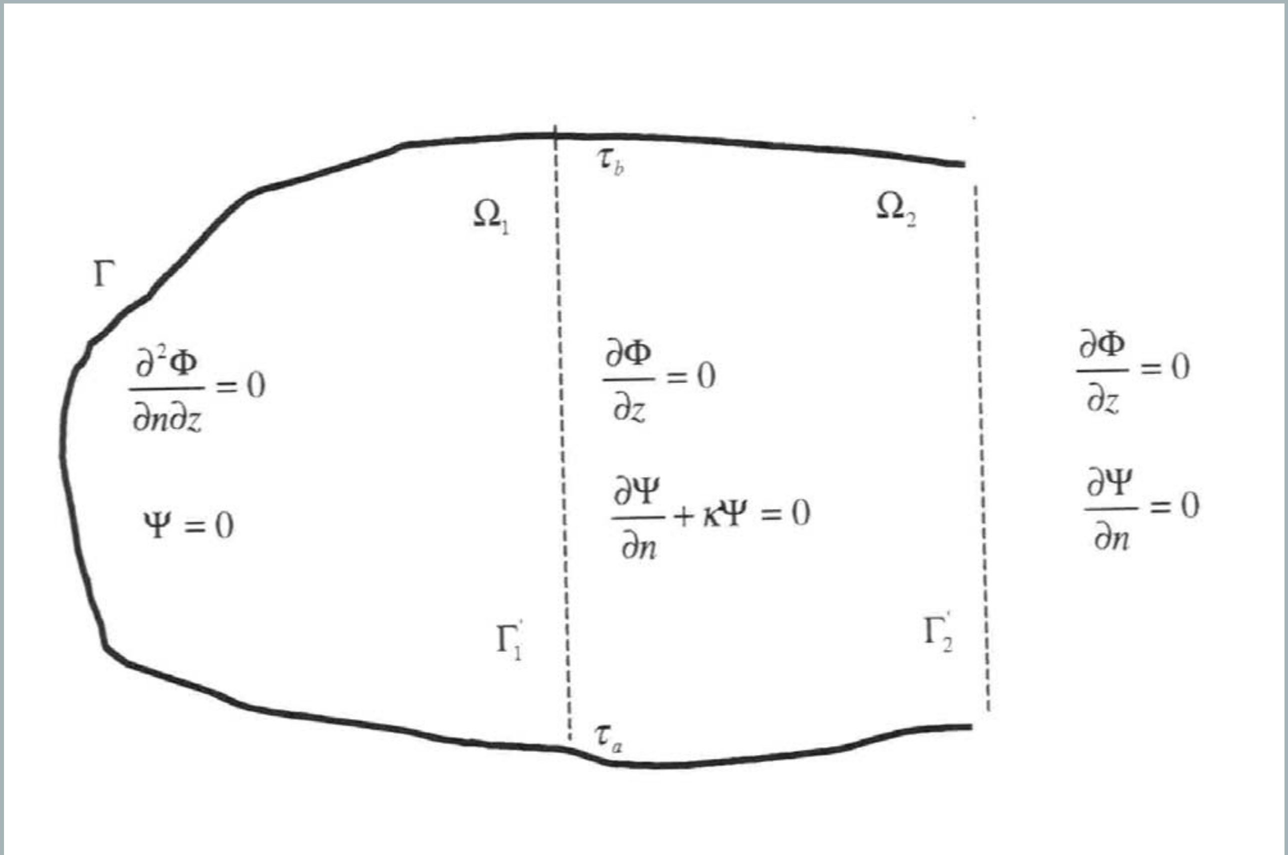
$$\mathbf{u} = \boldsymbol{\nabla} \times (\mathbf{r}\Psi) + \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times (\mathbf{r}\Phi) \, .$$

Flow Decomposition

$$u = \frac{\partial \Psi}{\partial y} + \frac{\partial^2 \Phi}{\partial x \partial z}, \qquad v = -\frac{\partial \Psi}{\partial x} + \frac{\partial^2 \Phi}{\partial y \partial z},$$

- $\nabla^2 \Psi = - \zeta$, ζ is relative vorticity
- $\nabla^2 \Phi = - w$

Boundary Conditions



Basis Functions

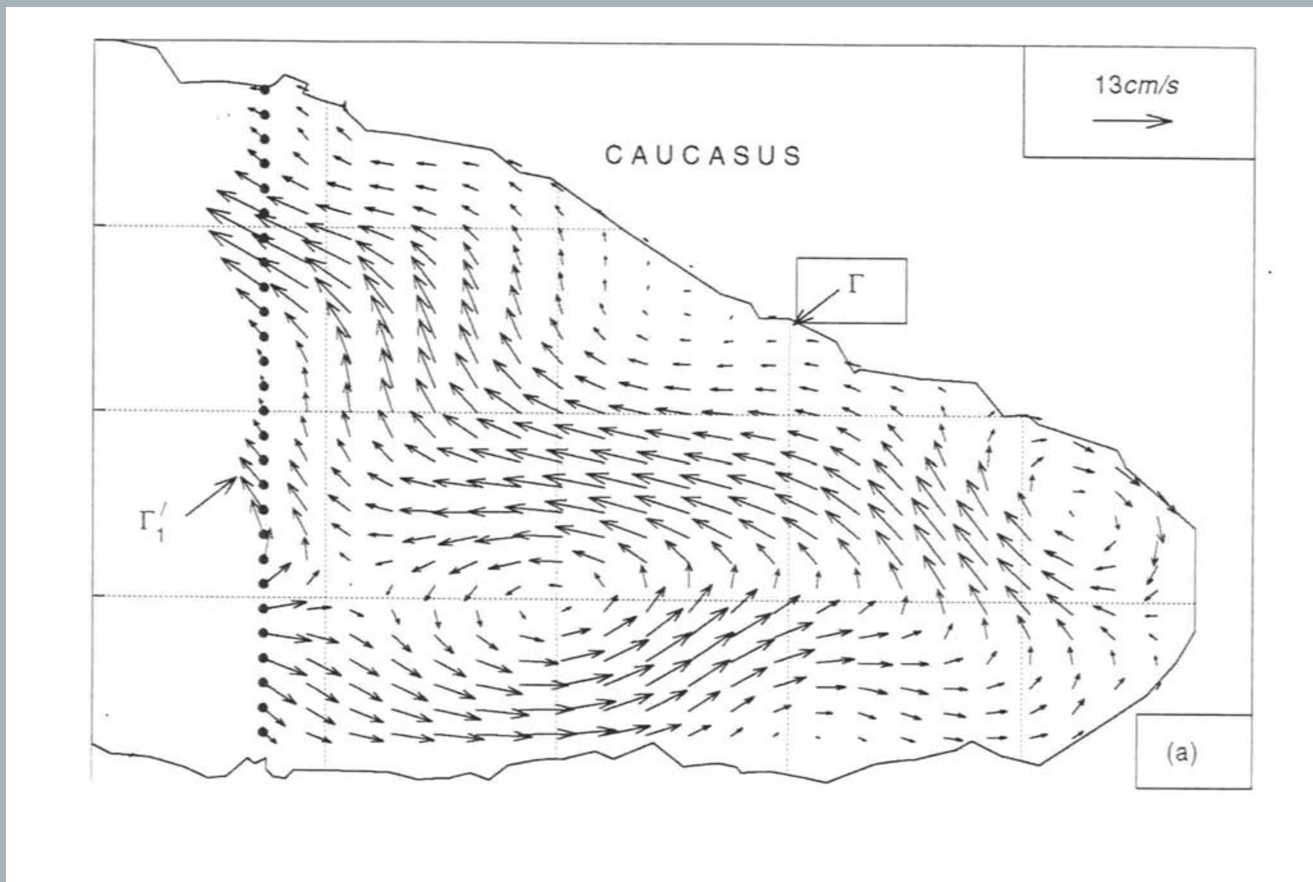
$$\Psi(x,y,z,t^\circ)=\sum_{k=1}^{\infty}a_k(z,t^\circ)\Psi_k(x,y,z,\kappa^\circ),$$

$$\frac{\partial\Phi(x,y,z,t^\circ)}{\partial z}=\sum_{m=1}^{\infty}b_m(z,t^\circ)\Phi_m(x,y,z),$$

Flow Reconstruction

$$
\begin{aligned}
u_{KM} &= \sum_{k=1}^{K} a_k(z, t^\circ) \frac{\partial \Psi_k(x, y, z, \kappa^\circ)}{\partial y} + \sum_{m=1}^{M} b_m(z, t^\circ) \frac{\partial \Phi_m(x, y, z)}{\partial x}, \\
v_{KM} &= -\sum_{k=1}^{K} a_k(z, t^\circ) \frac{\partial \Psi_k(x, y, z, \kappa^\circ)}{\partial x} + \sum_{m=1}^{M} b_m(z, t^\circ) \frac{\partial \Phi_m(x, y, z)}{\partial y}
\end{aligned}
$$

Reconstructed Circulation



Several Comments

- *Reconstruction is a useful tool for processing real-time velocity data with short duration and limited-area sampling.*
- *The scheme can handle highly noisy data.*
- *The scheme is model independent.*
- *The scheme can be used for assimilating s*

(8) Turbulence/Wave Effects

(a) *Wave Momentum Flux in the Ocean (especially near the bottom)*

(b) *Surface Roughness Length*

(c) *Wave Effect on TKE*

Turbulence Parameterizations

- *Bulk Mixed Layer Models*
  - *Garwood (1977), Price et al. (1986), Chu et al. (1990), Chu and Garwood (1991), Chu (1993)*
- *Diffusion Models*
  - *Mellor and Yamada (1982)*
  - *Kantha and Clayson (1994)*

Flow Decomposition

<u>Description of waves and turbulence</u>

The describe the effect from waves and turbulence, define

$$u = U + \tilde{u} + u'$$

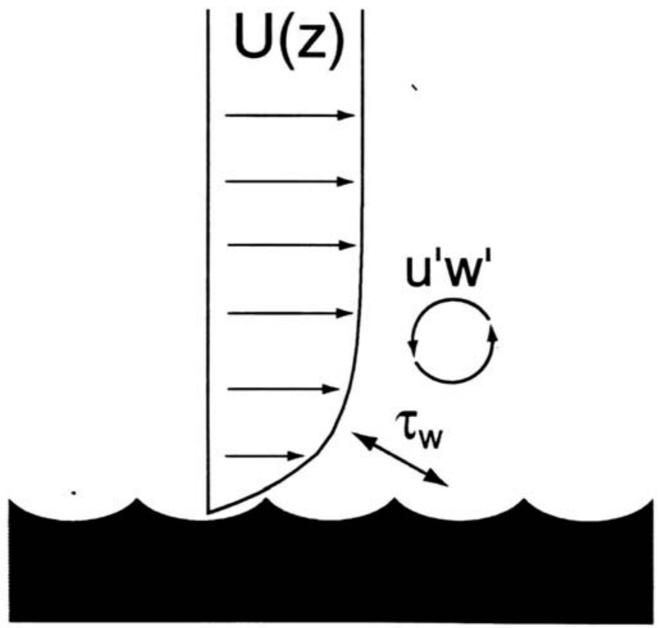
where

U-mean velocity,

$\tilde{u}$ -the wave induced (periodic) motions,

u'-turbulent fluctuations.

Momentum Flux



Equation for the momentum transfer

$$\frac{\partial U}{\partial t} = -\frac{\partial \langle u'w' \rangle}{\partial z} + \frac{\partial \tau_m^w}{\partial z}$$

Turbulence/Wave Effects

some simplifications (boundary layer etc)…

$$\frac{\partial U}{\partial t} + \ldots = \ldots - \frac{\partial}{\partial z}\left[\left\{\langle u'w'\rangle\right\} + \left\{\tilde{u}\tilde{w}\right\}\right] + \nu \frac{\partial^2 U}{\partial z^2}$$

$$\frac{\partial \tilde{u}}{\partial t} + \ldots = \ldots - \frac{\partial}{\partial z}\left[\langle u'w'\rangle\right] + \nu \frac{\partial^2 \tilde{u}}{\partial z^2}$$

$$\frac{\partial \langle u'w'\rangle}{\partial t} + \ldots = .. + F\left[U(t,z),\ \tilde{u}(\tilde{t},z)\right]$$

where

{ } represent an average over the wave motions

<> is average over turbulent timescales (short)

Wave Stress in the Interior

Wave stress in the interior

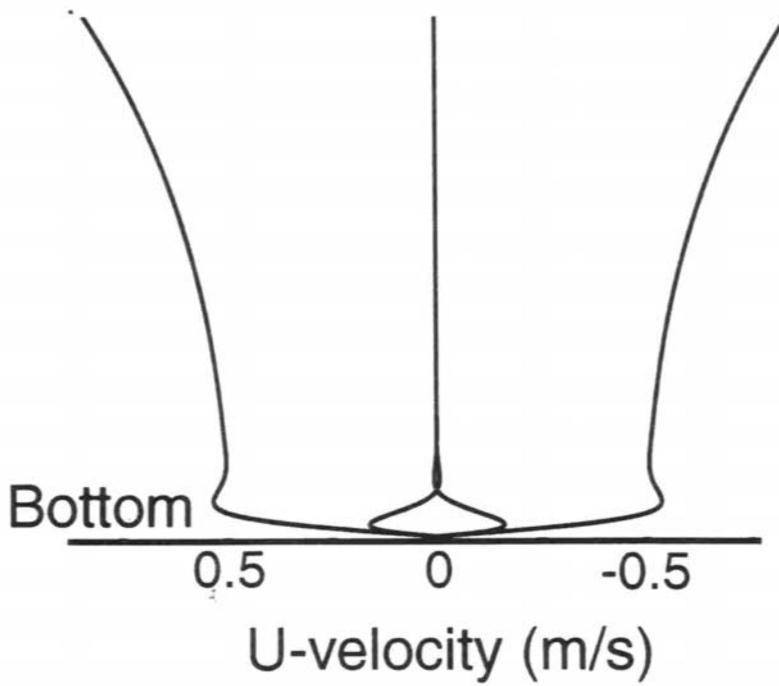
Waves are irrotational, thus, in the interior.

$$\frac{\partial\{\tilde{u}\tilde{w}\}}{\partial z} \equiv 0$$

Bottom Boundary Layer

Wave stress in the bottom boundary layer

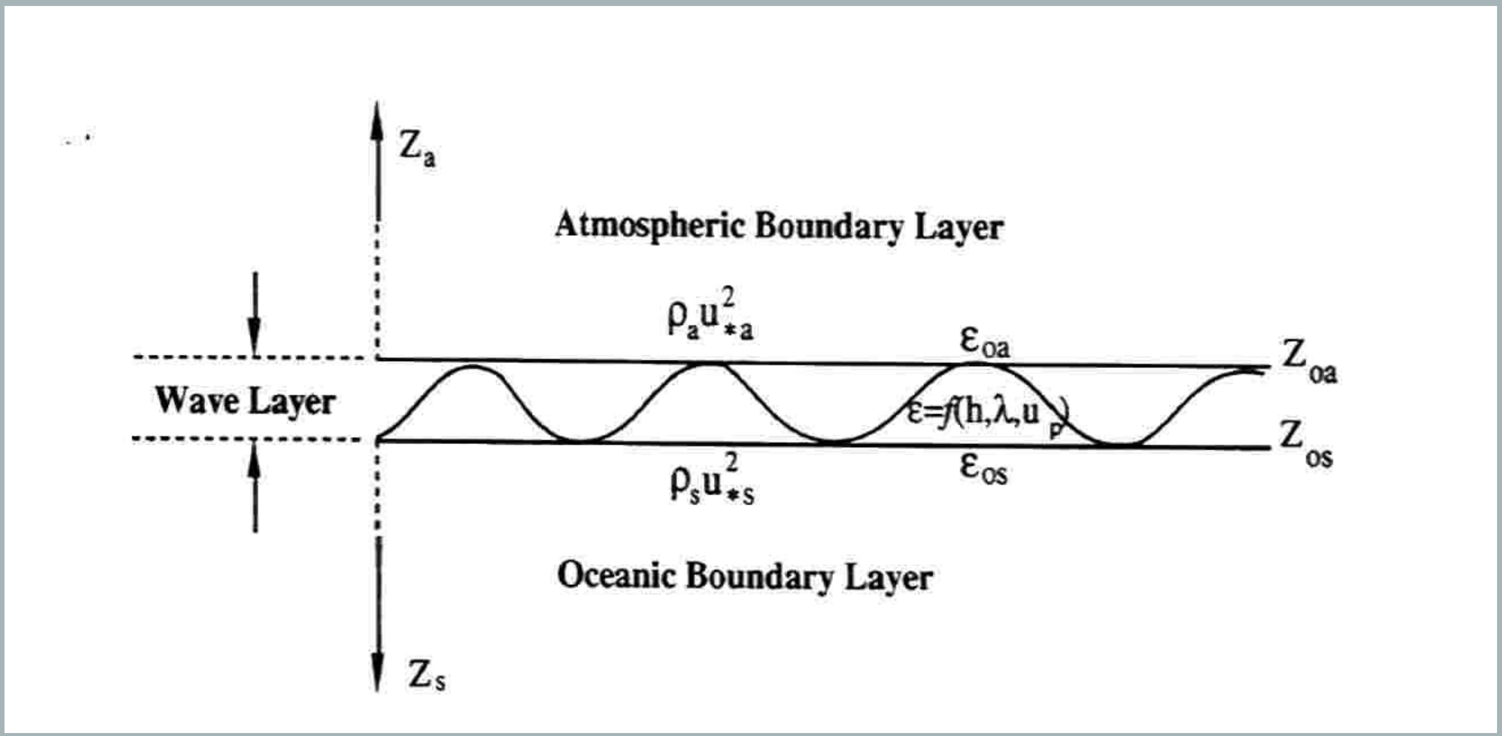
Close to the bottom, friction is important which effects the wave velocity field



$$\frac{\partial\{\tilde{u}\tilde{w}\}}{\partial z} \equiv 0$$

$$\frac{\partial\{\tilde{u}\tilde{w}\}}{\partial z} \neq 0$$

Wave Effect on Surface Roughness Length



- *TKE Dissipation in Air and Ocean are Functions of Wave Variables*
- $\varepsilon = f(h, \lambda, u_{ph})$

Wave Breaking & Turbulent Dissipation

- *TKE Equation:*
- $D\ TKE/DT = S + WB \pm B - D$
  - *S: Shear Production*
  - *WB: Wave Breaking Effect*
  - *B: Buoyancy Production or Damping*
  - *D: Dissipation*
- $$WB = \gamma\ (u_{ph}h)^3/\ \lambda^4$$