Ensemble Mine Impact Burial Prediction

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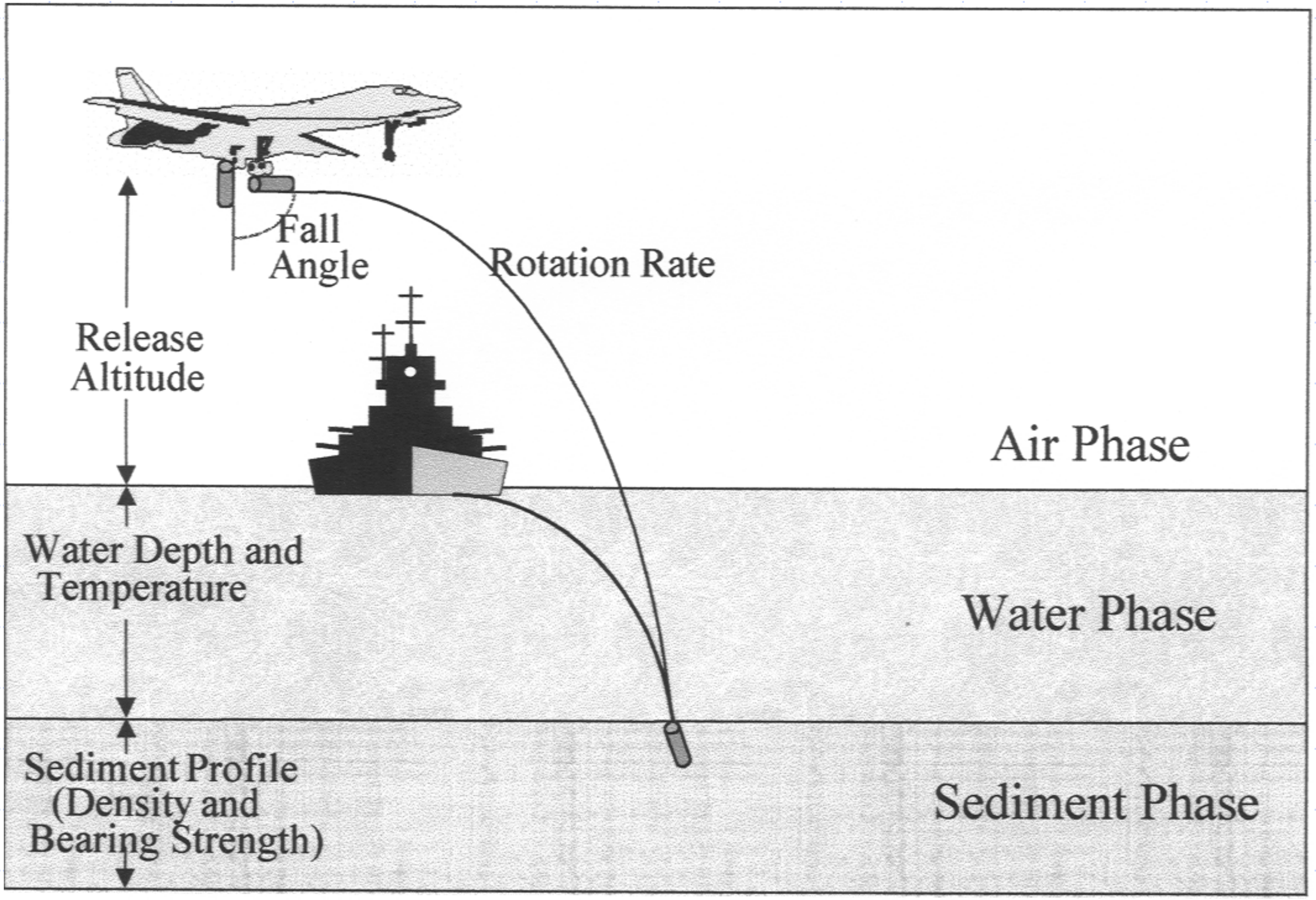
Typical Mine Insertion Profile



Modeling Mine Impact Burial Depth

- Modeling is first step in planning mine hunting mission
- Determining depth of burial, and height, area and volume protruding upon impact is first step in modeling the mine's situation

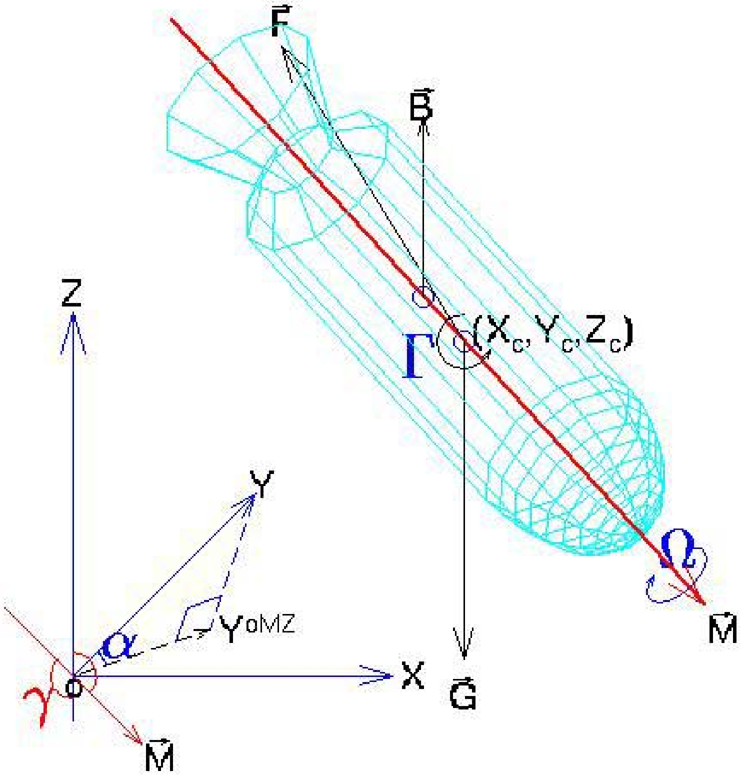
The Center of Gravity

- $x_c = \int x\, dm / \int dm$
- $y_c = \int y\, dm / \int dm$
- $z_c = \int z\, dm / \int dm$

Six Parameters for Determination of Mine Position

- Coordinate of Center of Gravity (x_c, y_c, z_c)
- Direction Angles with Cartesian Coordinate (ϕ_x, ϕ_y, ϕ_z)

Six Parameters for Determination of Mine Position



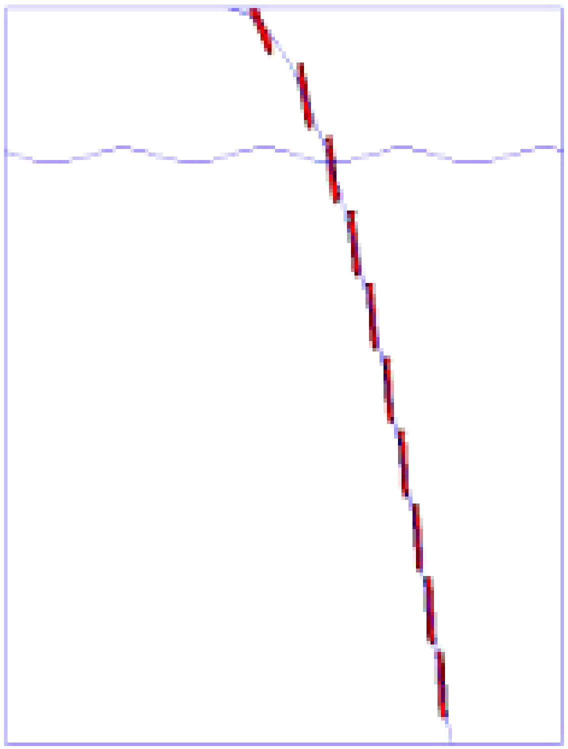
Development of Navy's Impact Burial Prediction Model (IBPM)

- Arnone & Bowen Model (1980) – Without Rotation
- Modified Impact Burial Model (Satkowiak, 1987-88) – With Rotation
- IMPACT25/28 (Hurst, 1992) –
- Environmental Impact on IMPACT25 (Chu et al., 1999, 2000, 2002; Taber 1999, Smith 2000, Gilles 2001)

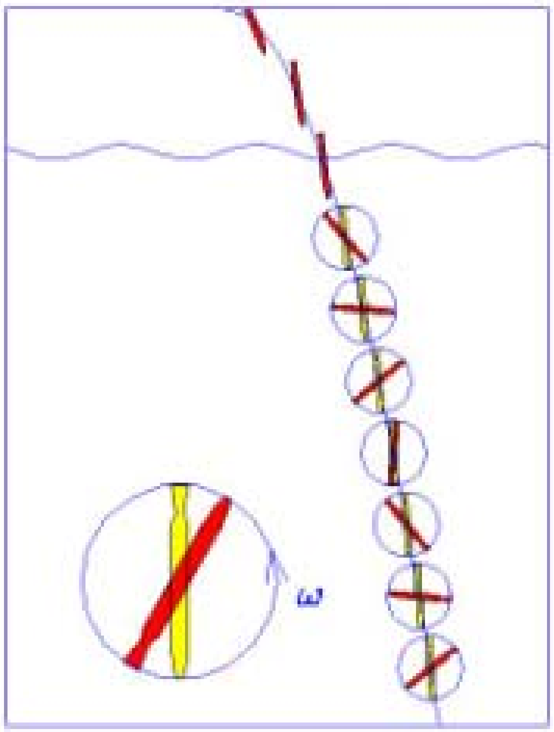
Problems in IMPACT25/28

- **Hydrodynamics**
- **Sediment**

Hydrodynamic Theory in IMPACT25/28



Without Rotation
Without Moment Equation



With Rotation but
Without Moment Equation

Sediment Phase (IMPACT25/28)

- $\int a \, dm = B + D + P$
- B = *Bearing Strength (~75%)*
- D = *Hydrodynamic Drag (~25%)*
- P = *Buoyancy (~5%)*

Bearing Strength *(IMPACT25/28)*

- $B = Nc$ *Shear Stress,* $Nc \sim 9 - 11$

  Shear Stress data SS(z) needed

- $\boldsymbol{B} = \boldsymbol{B}_0 \, (SR)^{0.15}$

  SR = *Strain Rate,*

  $\boldsymbol{B}_0$ = *Bearing Strength at unity strain rate*

Hydrodynamic Drag in Sediment

- $D = C_D \ \ A V^2/2$

$A =$ *Mine cross-section area*

$=$ *Sediment density*

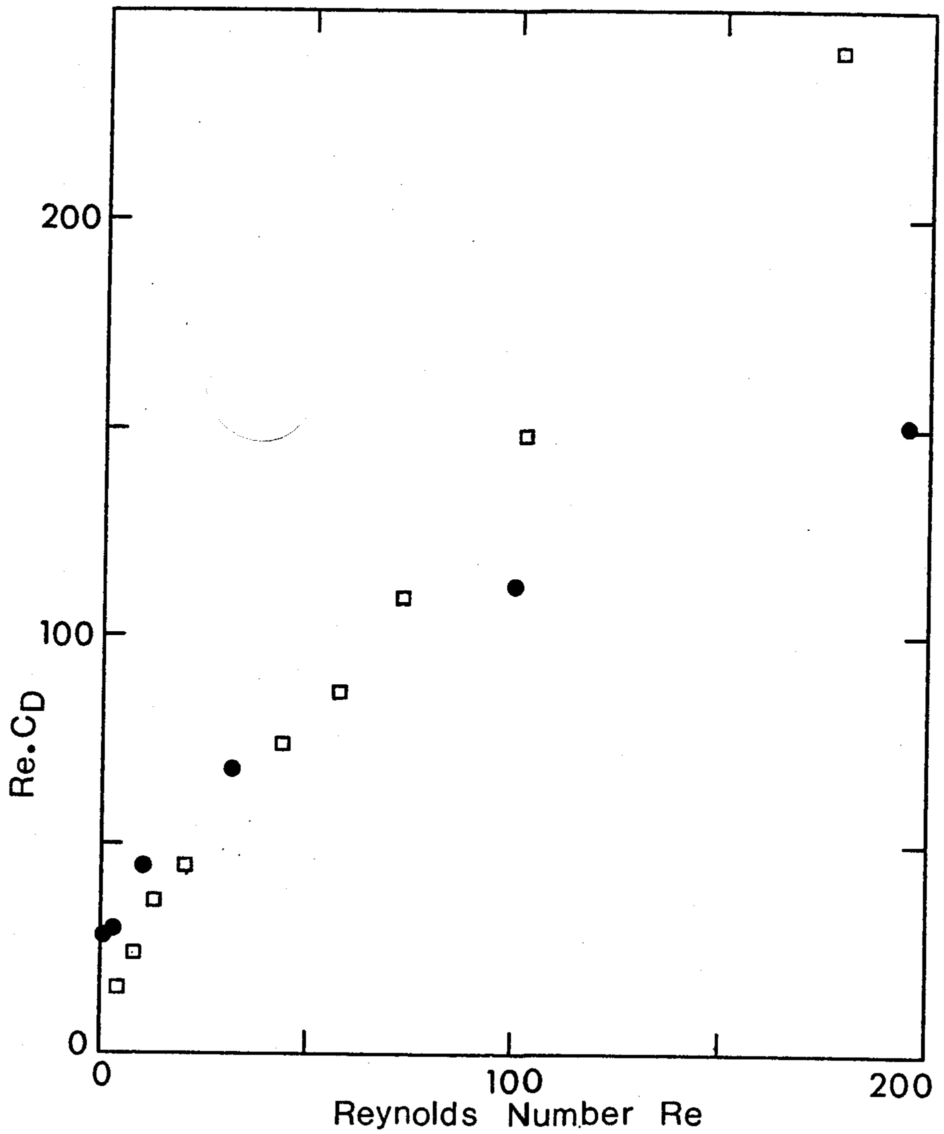
$V =$ *Mine speed*

$C_D =$ *Drag Coefficient* $= (2C_1 + C_2 Re)/Re$

$Re = V^2/SS$, $SS =$ *Sediment Shear Strength*

Data of $SS(z)$ *needed.*

C_D (Sediment)



Buoyancy (Sediment)

- $P = gh\,(\quad - \quad {}_{w})$
- h = depth
- ${}_{w}$ = Water Density

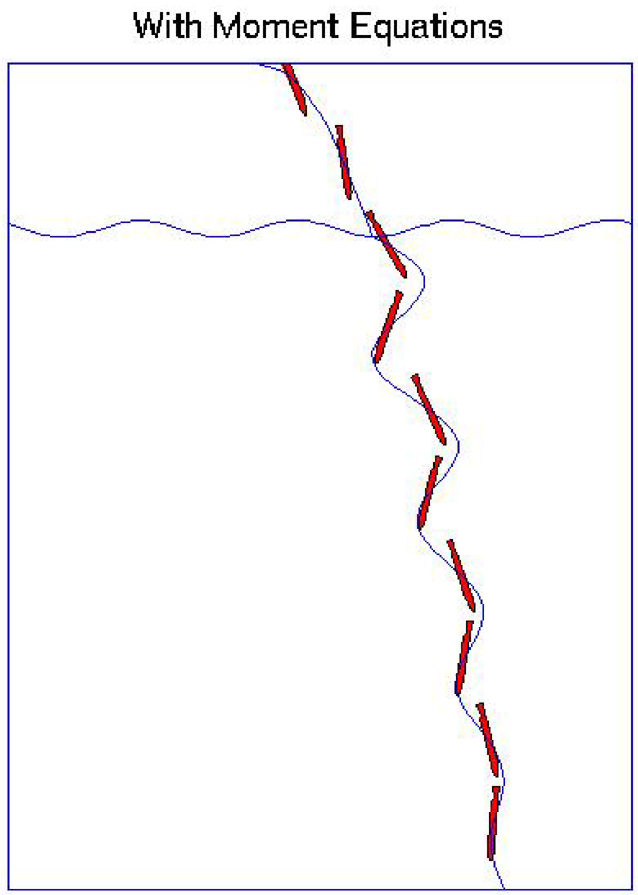
- Sediment density profile data needed:

  (z)

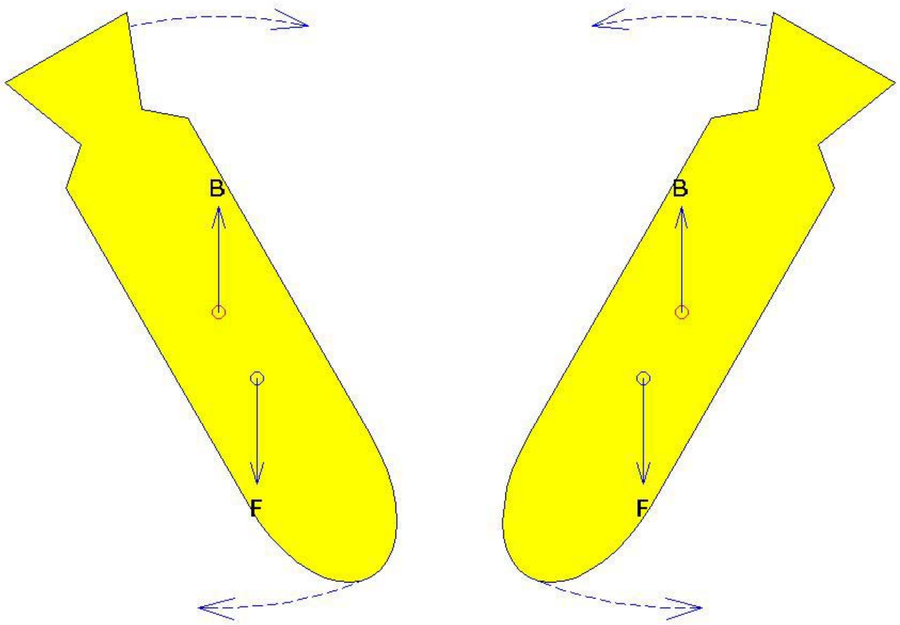
Is the hydrodynamics of mine impact realistic?

No.

Spiral-Type Motion of Mine



Spiral-type motion of mine



Six Parameters for Determination of Mine Position

- Coordinate of Center of Gravity (x_c, y_c, z_c)
- Direction Angles with Cartesian Coordinate (ϕ_x , ϕ_y, ϕ_z)

Hydrodynamic Theory

- Solid Body Falling Through Fluid Should Obey 2 Physical Principles:

1. <u>Momentum Balance</u>

$$\int (dV^* / dt^*) dm^* = W^* + F_b^* + F_d^*$$

2. <u>Moment of Momentum Balance</u>

$$\int [r^* \times (dV^* / dt^*)] dm^* = M^*$$

* Denotes dimensional variables

V^* → Velocity

W^* → gravity

F_b^* → buoyancy force

F_d^* → drag force

M^* → resultant moment

Sensitivity Studies on IMPACT25/28

- Environmental Sensitivity Study (Chu et al. 1999, Taber 1999)
- Mine Impact Burial Experiment (MIBEX) at Monterey Bay (Chu et al. 2000, Smith 2000)
- Mine Drop Experiment (MIDEX) (Chu et al. 2001, 2002; Gilles 2001)

Environmental Sensitivity Study (Chu et al. 1999)

- Hydrodynamics and sedimentation are key factors to affect the mine impact burial.

IMPACT25/28 Model Input Parameters

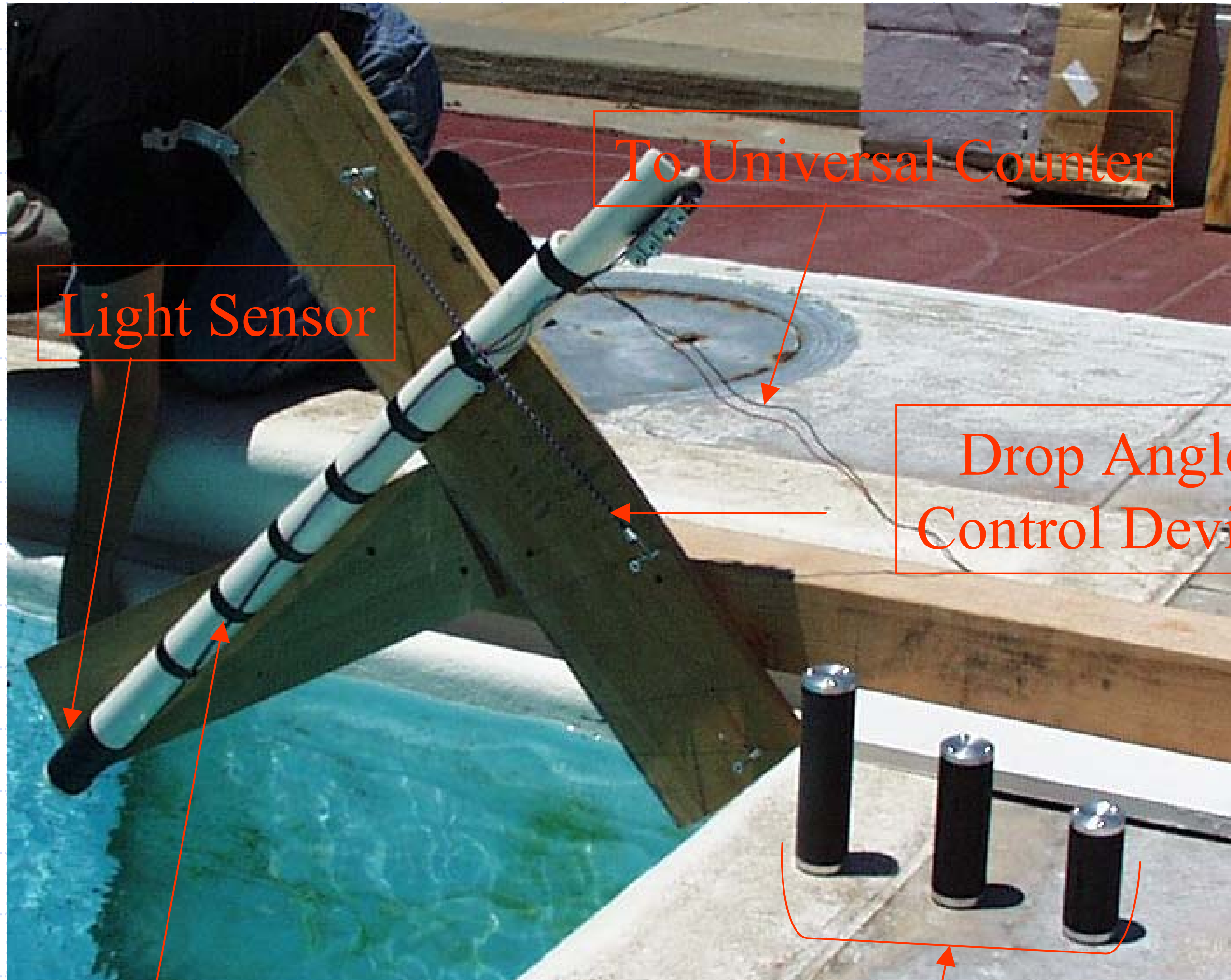
- Mine Parameters
  - Mass in Air
  - Mass in Water
  - Length
  - Diameter
  - Maximum diameter
  - CM displacement
- Altitude when released
- Angle when released
- Initial horizontal and vertical velocity
- Initial Rotation rate
- Water depth
- Water Temperature
- Sediment Parameters
  - Density
  - Shear Strength

MIDEX at NPS (May 3, 2001)

- Proposal was to conduct several controlled “mine” drops in real world environment while simultaneously gathering sediment data and oceanographic data to determine effect on code output.
- Synchronized environmental and mine burial data

MIDEX (Small Scale)

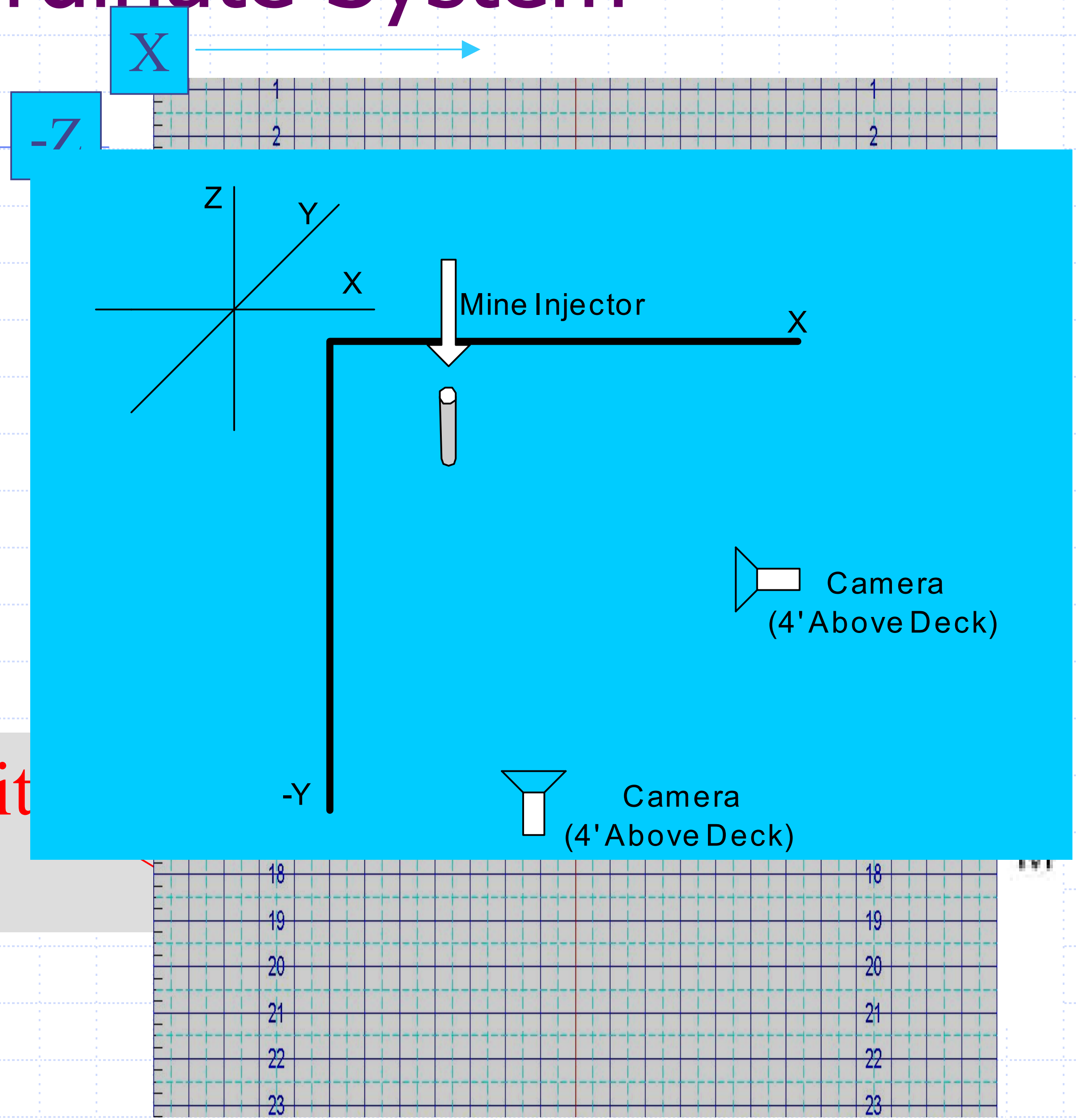
- MIDEX designed to examine the uniform density assumption of IMPACT 25, namely what effect a varying center of mass will have on a mine shape's water phase trajectory.
- Controlled Parameters:
  1. Drop Angles: 15º, 30º, 45º, 60º, 75º.
  2. Center of Mass Position.
  3. L/D ratio (constant).
  4. V_{init} (to some extent).
- Conducted several tests for each drop angle, center of mass position and initial velocity.



Mine Injector

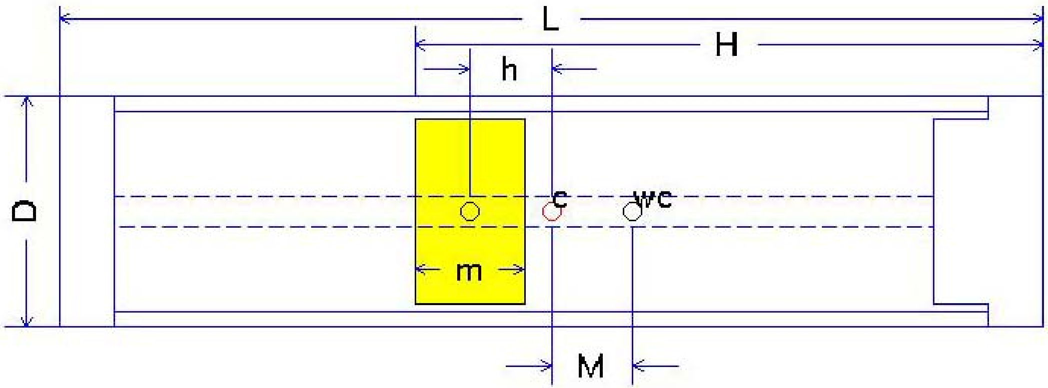
Mine Shapes:
Length: 15, 12, 9 cm
Diameter: 4 cm

Coordinate System



Mine Attit
(Psi)

Center of Mass



MODEL # 1

L=15.1359cm D=4cm m=2.7cm

Weight=322.5 g Volume=190.2028 cm³ Density=1.6956 g/cm³

H:	10.380	8.052	5.725	cm
h:	−1.462	0.866	3.193	cm
M:	0.000	18.468	36.935	mm

MODEL # 2

L=12.0726cm D=4cm m=1.7cm

Weight=254.2 g Volume=151.709 cm³ Density=1.6756 g/cm³

H:	8.450	6.609	4.768	cm
h:	−1.564	0.277	2.119	cm
M:	0.000	12.145	24.290	mm

MODEL # 3

L=9.1199cm D=4cm m=1.47cm

Weight=215.3 g Volume=114.6037 cm³ Density=1.8786 g/cm³

H:	6.662	5.592	4.521	cm
h:	−1.368	−0.297	0.774	cm
M:	0.000	6.847	13.694	mm

Defined COM position as:
2 or -2: Farthest from volumetric center
1or -1
0: Coincides with volumetric center

Hydrodynamic Theory

- Considering both momentum and moment of momentum balance yields 9 governing equations that describe the mine's water phase trajectory.

$$\frac{dV_1}{dt}+\omega_2 V_3-\omega_3 V_2=-\frac{C_D\rho_w}{2\rho_m}|\vec{V}|(V_1-V_{w1})+\frac{\rho_m-\rho_w}{\rho_m}\cos\psi_1$$

$$\frac{dV_2}{dt}+\omega_3 V_1-\omega_1 V_3=-\frac{C_D\rho_w}{2\rho_m}|\vec{V}|(V_2-V_{w2})+\frac{\rho_m-\rho_w}{\rho_m}\cos\psi_2$$

$$\frac{dV_3}{dt}+\omega_1 V_2-\omega_2 V_1=-\frac{C_D\rho_w}{2\rho_m}|\vec{V}|(V_3-V_{w3})+\frac{\rho_m-\rho_w}{\rho_m}\cos\psi_3$$

$$J_1\frac{d\omega_1}{dt}+(J_3-J_2)\omega_2\omega_3-J_{31}(\frac{d\omega_3}{dt}+\omega_1\omega_2)=\frac{LM_1^*}{g}$$

$$J_2\frac{d\omega_2}{dt}+(J_1-J_3)\omega_3\omega_1-J_{31}(\omega_3^2-\omega_1^2)=\frac{LM_2^*}{g}$$

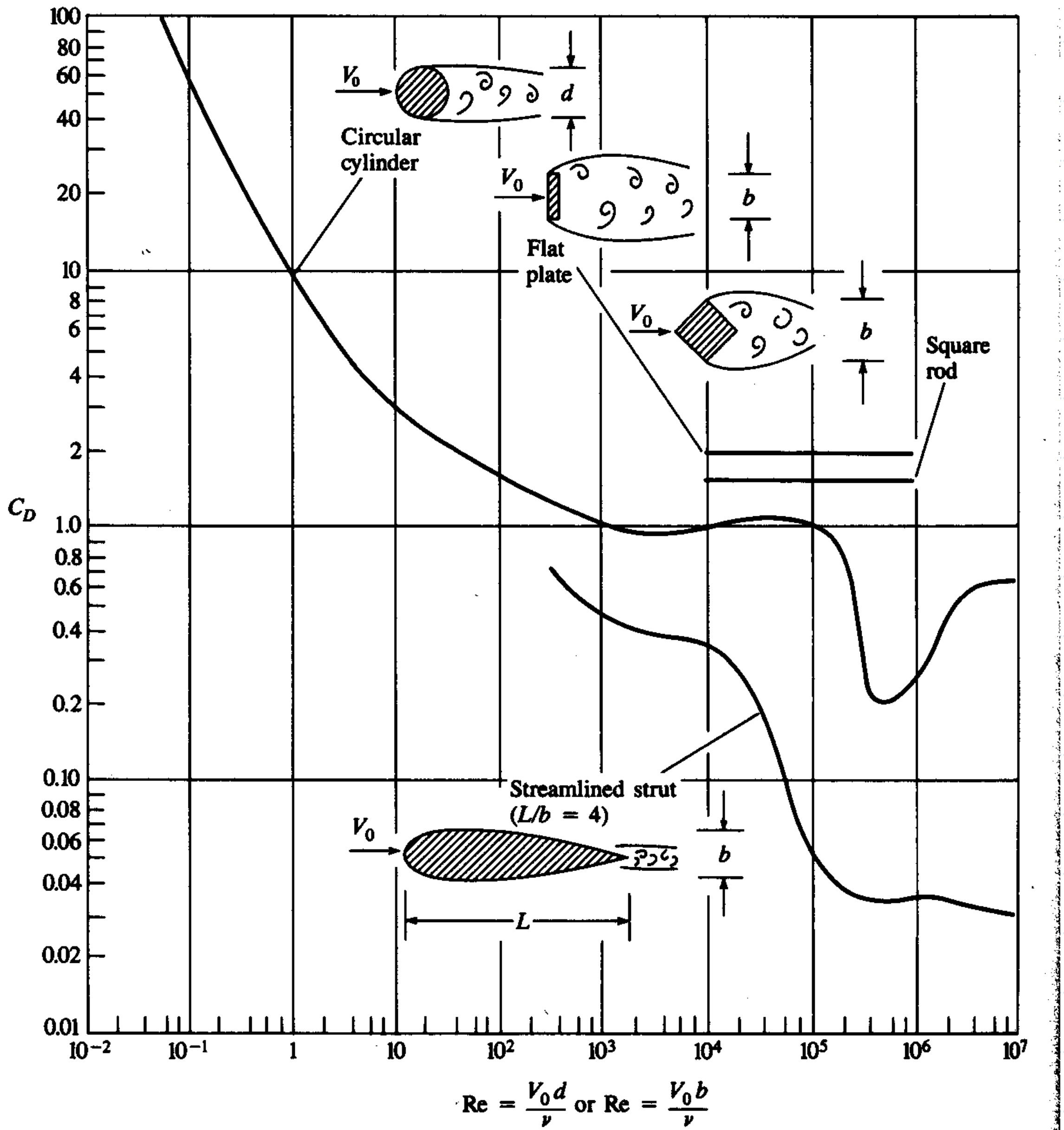
$$J_3\frac{d\omega_3}{dt}+(J_2-J_1)\omega_1\omega_2-J_{31}(\frac{d\omega_1}{dt}-\omega_2\omega_3)=\frac{LM_3^*}{g}$$

$$\frac{d}{dt}\cos\psi_1=\omega_3\cos\psi_2-\omega_2\cos\psi_3$$

$$\frac{d}{dt}\cos\psi_2=\omega_1\cos\psi_3-\omega_3\cos\psi_1$$

$$\frac{d}{dt}\cos\psi_3=\omega_2\cos\psi_1-\omega_1\cos\psi_2$$

Dependence of Drag Coefficient on Reynolds Number (Roberson & Crowe, 1990)



Reynolds Number

- MIDEX (NPS): $Re \sim 10^4$
- Carderock Test: $Re \sim 8 \quad 10^4$
- MIDEX data set is useful for model update and validation.

Data Analysis

1. Video converted to digital format.
2. Digital video from each camera analyzed frame by frame (30Hz) using video editing program.
3. Mine’s top and bottom position determined using background x-z and y-z grids. Positions manually entered into MATLAB for storage and later processing.
4. Analyzed 2-D data to obtain mine’s x,y and z center positions, attitude (angle with respect to z axis) and u,v, and w components.

Non-dimensional Conversions

- In order to generalize results, data was converted to non-dimensional numbers.

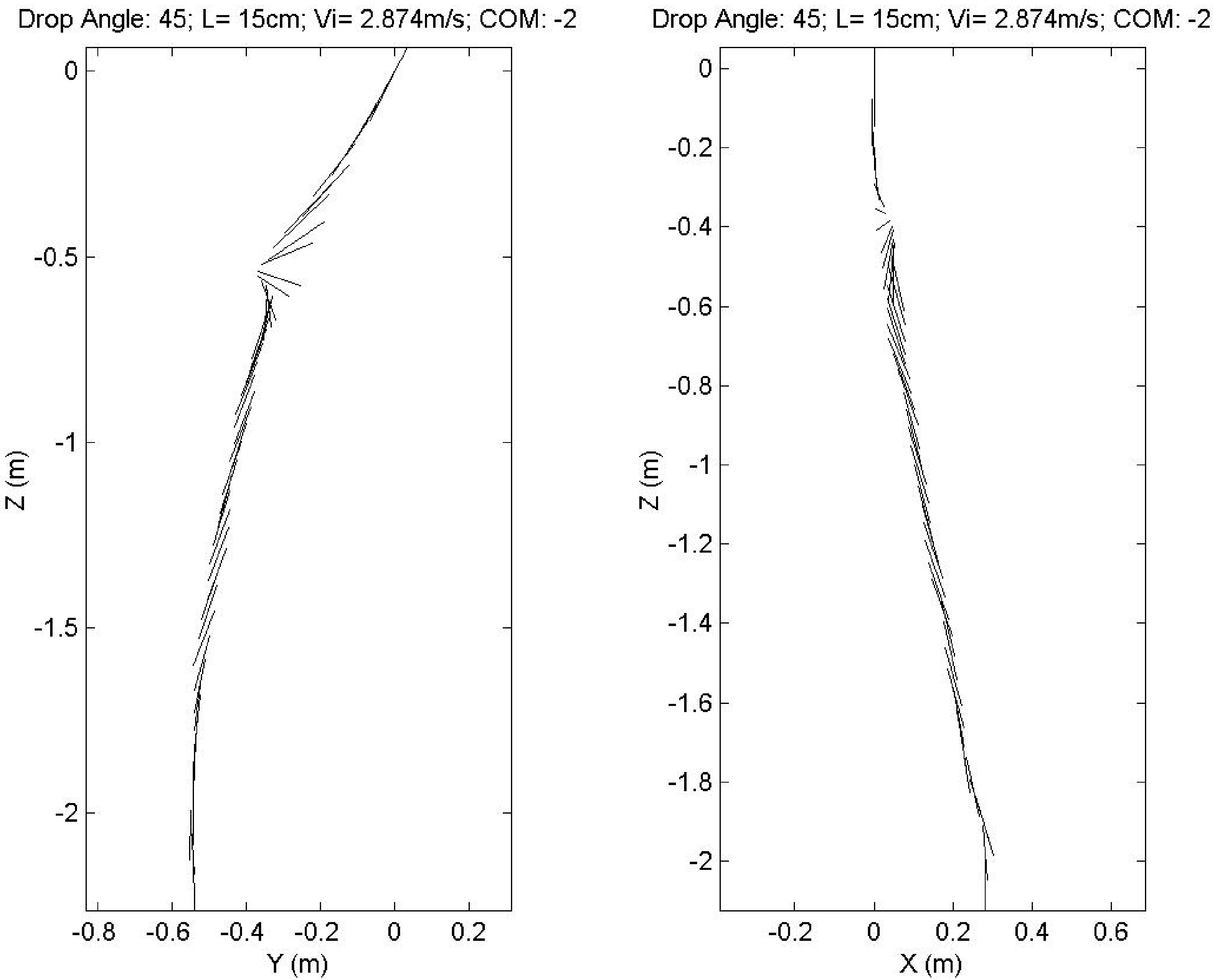
$$t^* = \frac{dt}{\sqrt{\frac{L}{g}}},\ V_i^* = \frac{V_i}{\sqrt{gL}},\ \frac{L}{D},\ COM = \frac{2\Delta L}{L},\ \frac{(x,y,z)}{L},\ \frac{(u,v,w)}{\sqrt{gL}}$$

Sources of Error

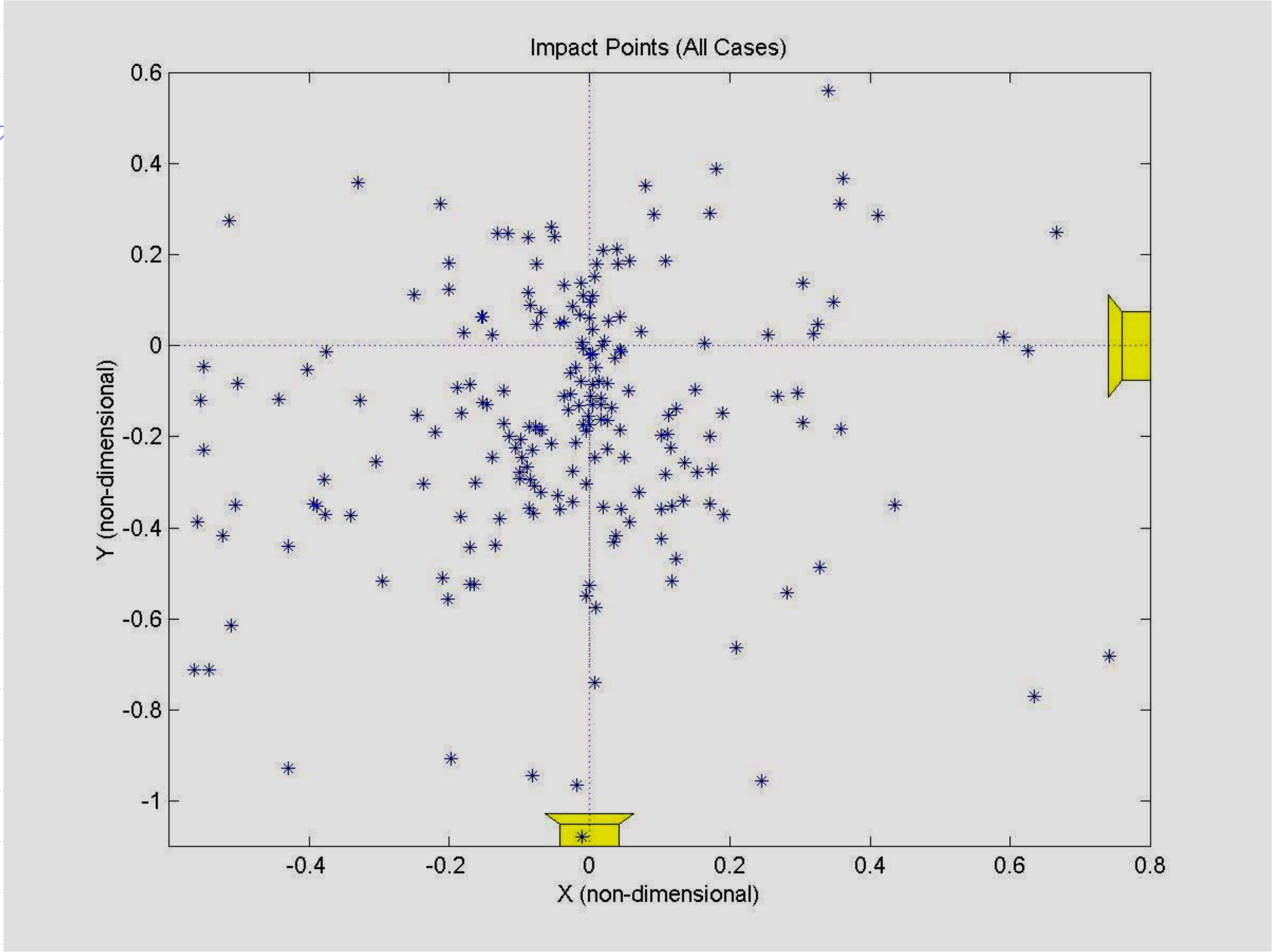
1. Grid plane behind mine trajectory plane. Results in mine appearing larger than normal.
2. Position data affected by parallax distortion and binocular disparity.
3. Air cavity affects on mine motion not considered in calculations.
4. Camera plane not parallel to x-y plane due to pool slope.

Underwater Video Clip

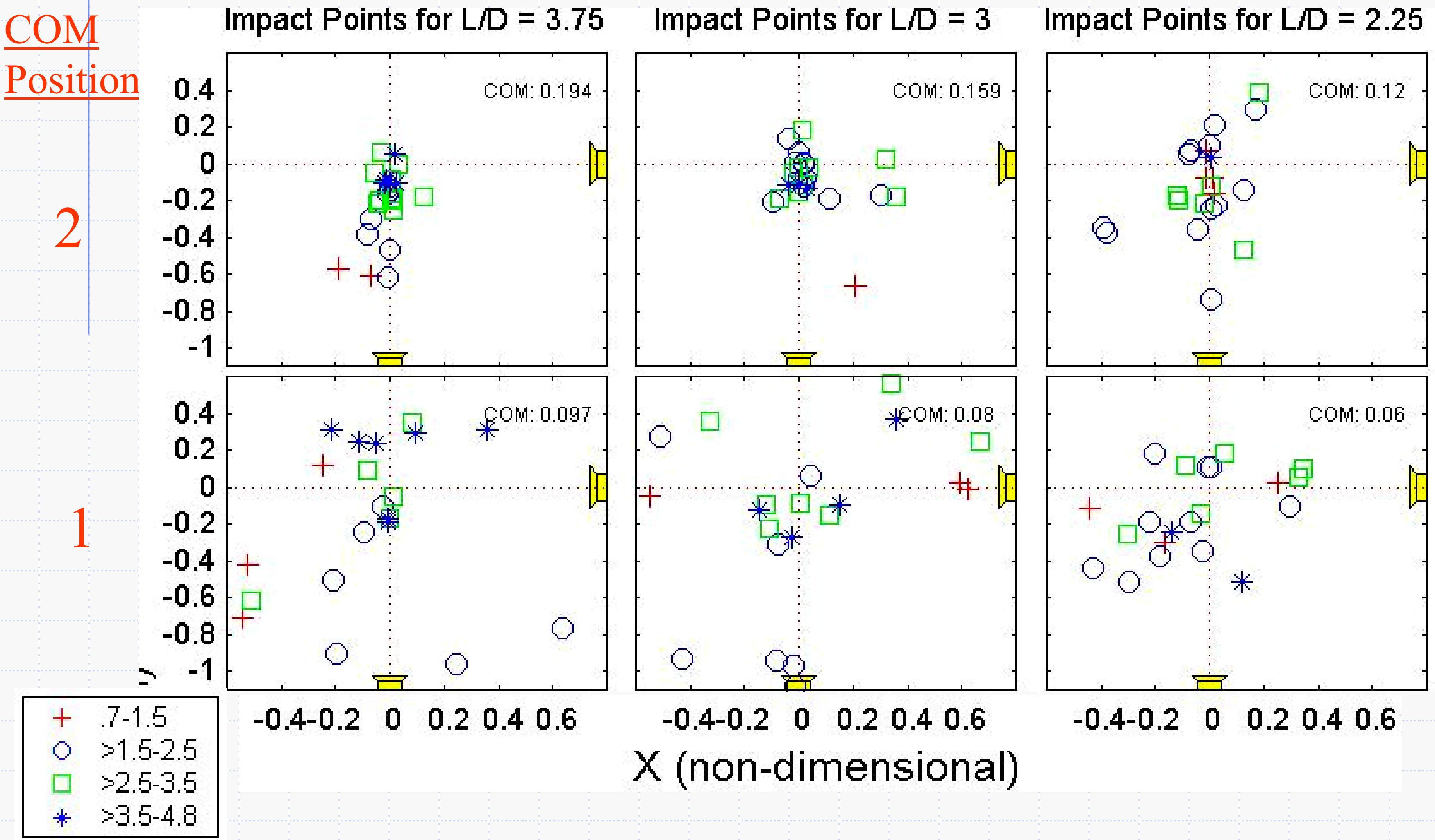
Center of Mass:
Position 2

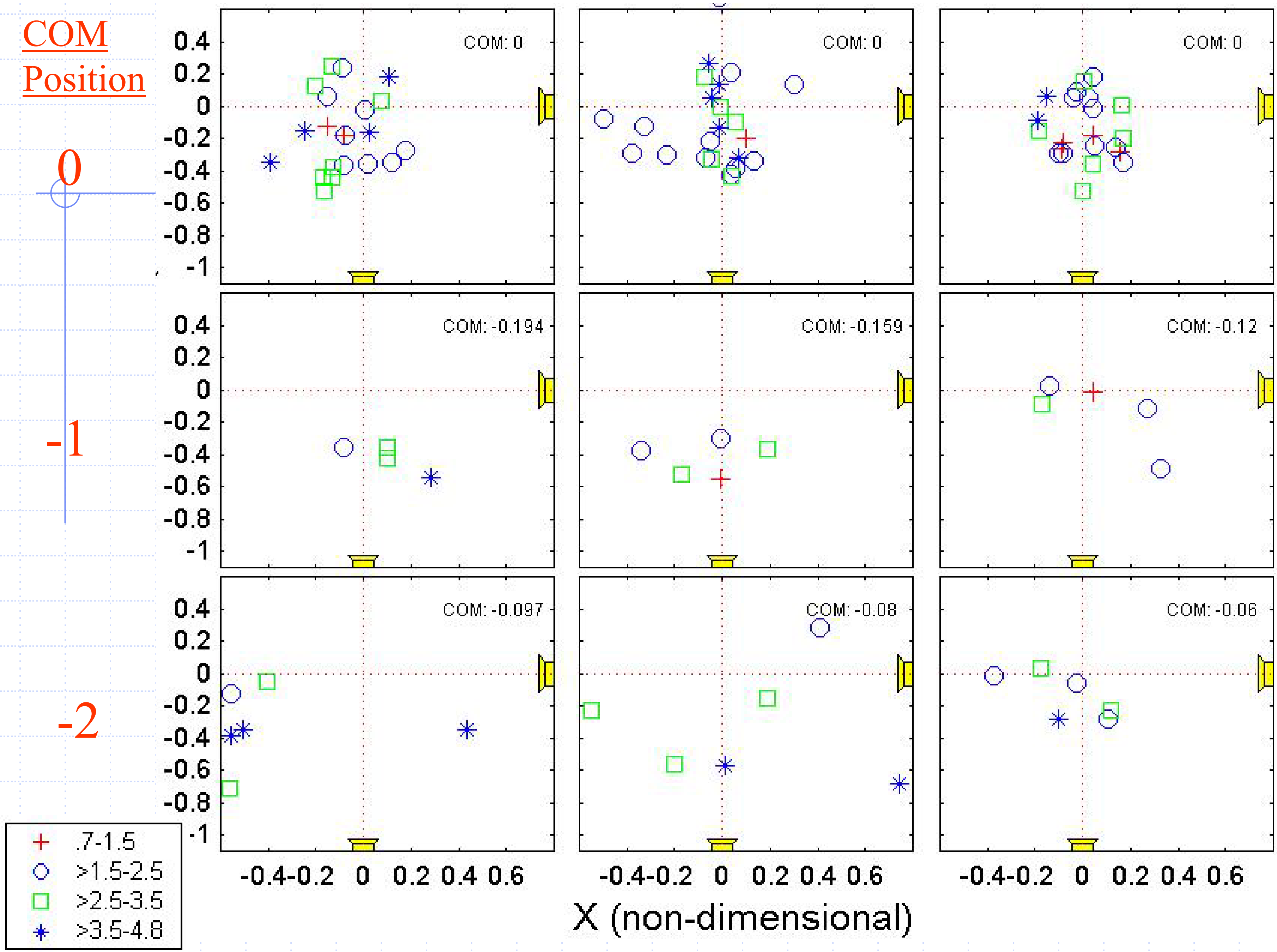

Drop Angle: 45; L= 15cm; Vi= 2.874m/s; COM: -2
0
-0.5
-1
-1.5
-2
Z (m)
-0.8
-0.6
-0.4
-0.2
0
0.2
Y (m)
Drop Angle: 45; L= 15cm; Vi= 2.874m/s; COM: -2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
Z (m)
-0.2
0
0.2
0.4
0.6
X (m)

Impact Point (All Cases)

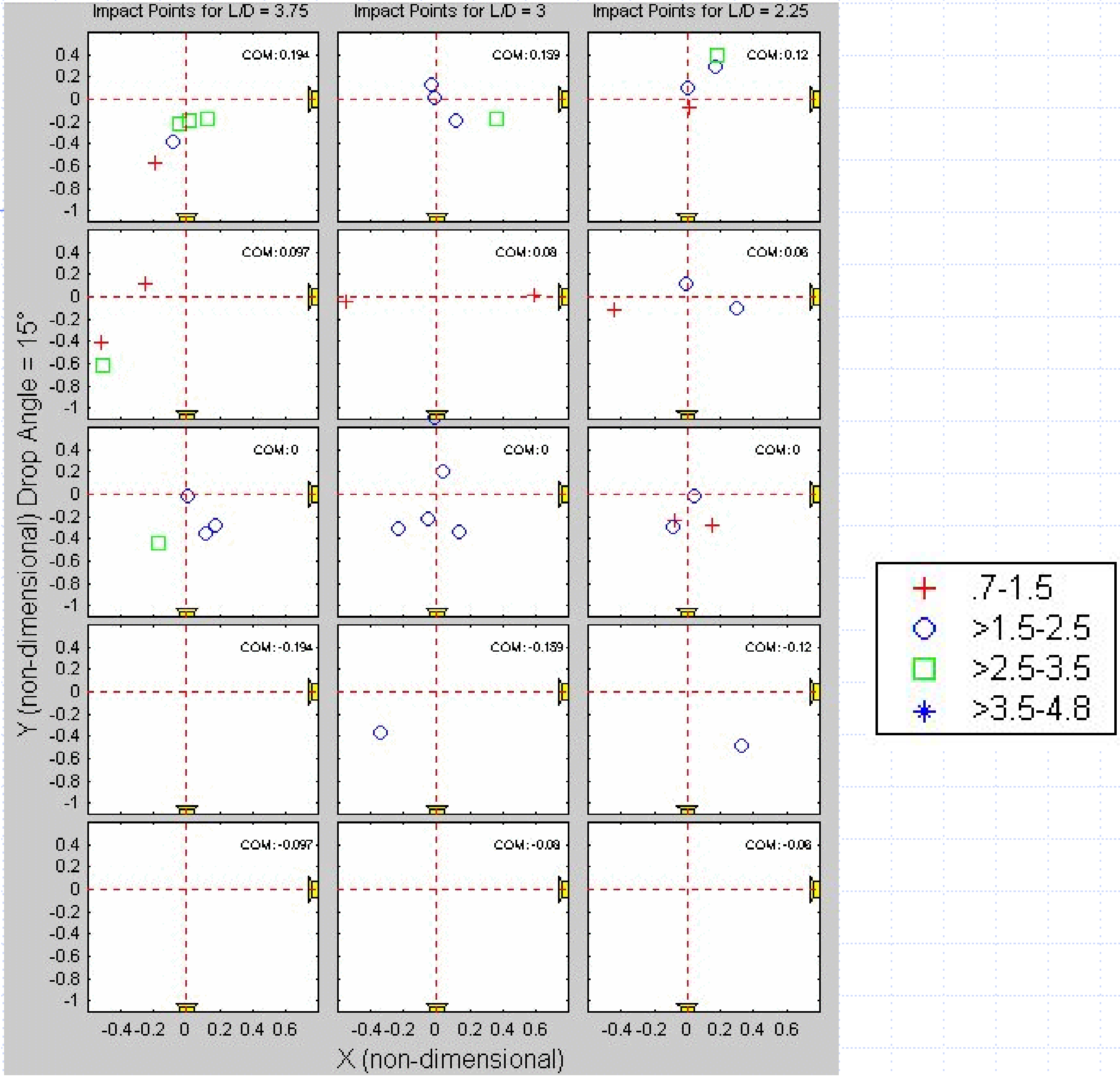


Impact Point (All Drop Angles)

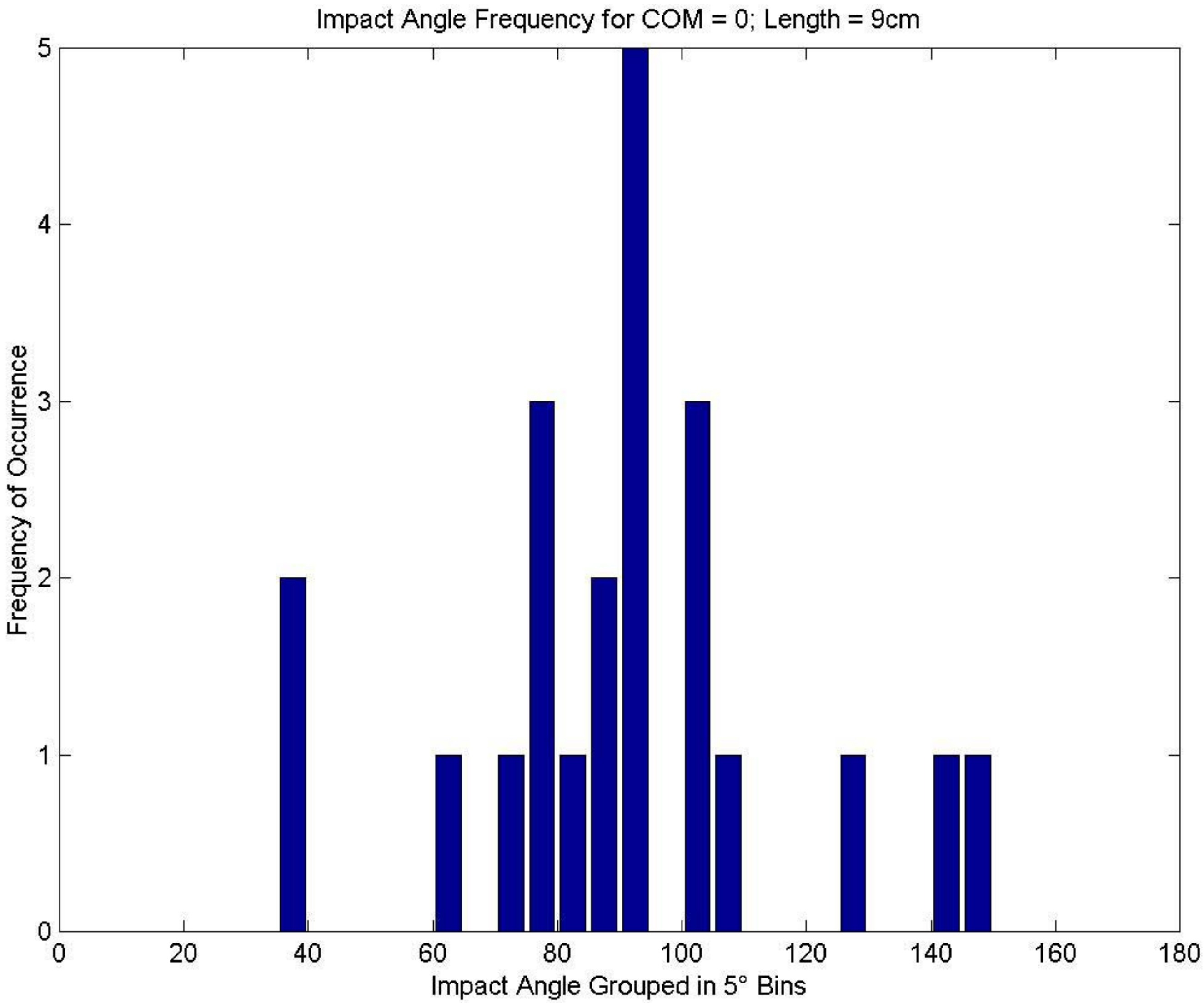



COM Position
0
-1
-2
COM: 0
COM: 0
COM: 0
COM: -0.194
COM: -0.159
COM: -0.12
COM: -0.097
COM: -0.08
COM: -0.06
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.4 -0.2 0 0.2 0.4 0.6
X (non-dimensional)
.7-1.5
>1.5-2.5
>2.5-3.5
>3.5-4.8

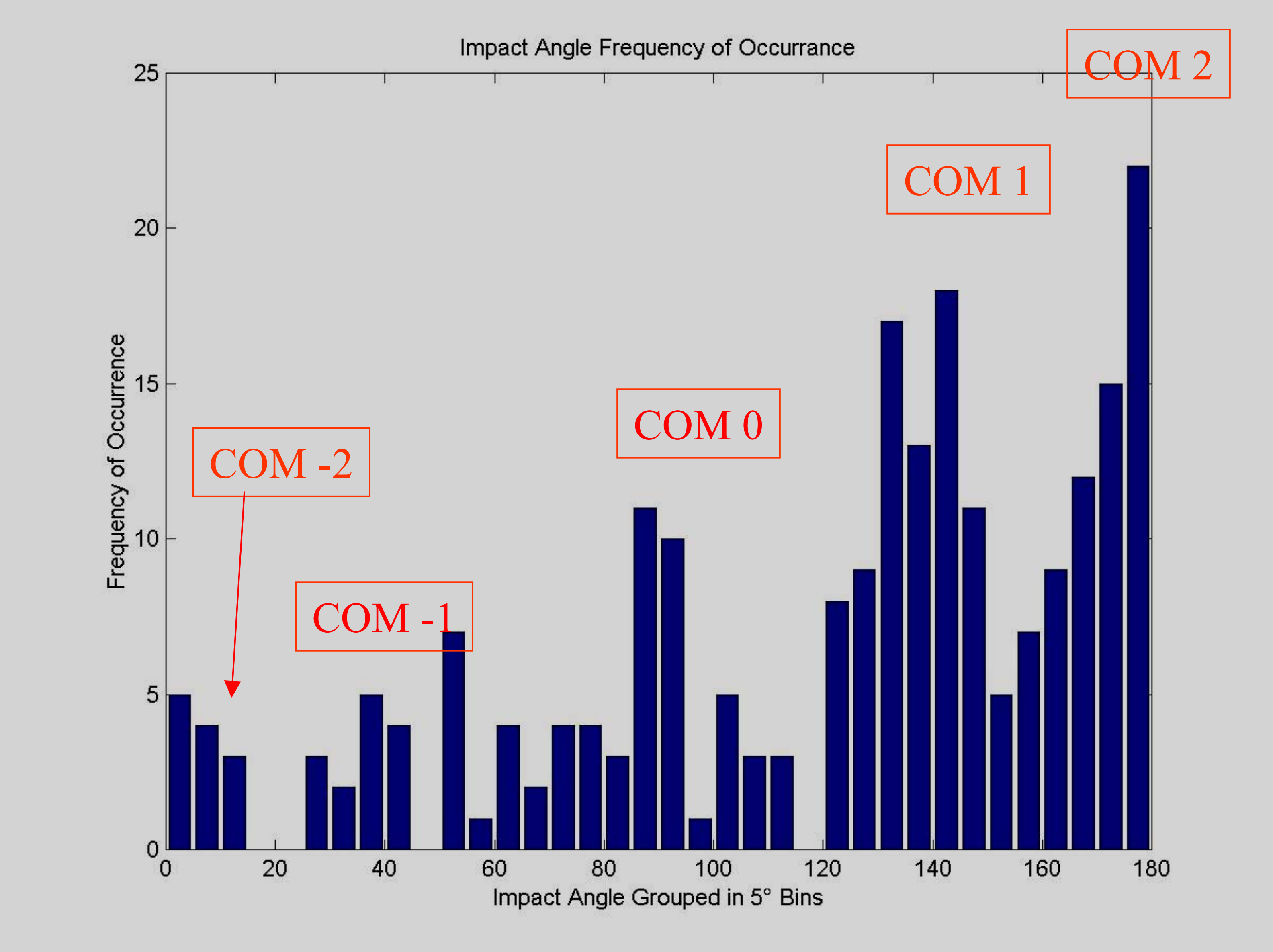
Impact Point (By Angle)



Impact Angle Frequency of Occurrence by L

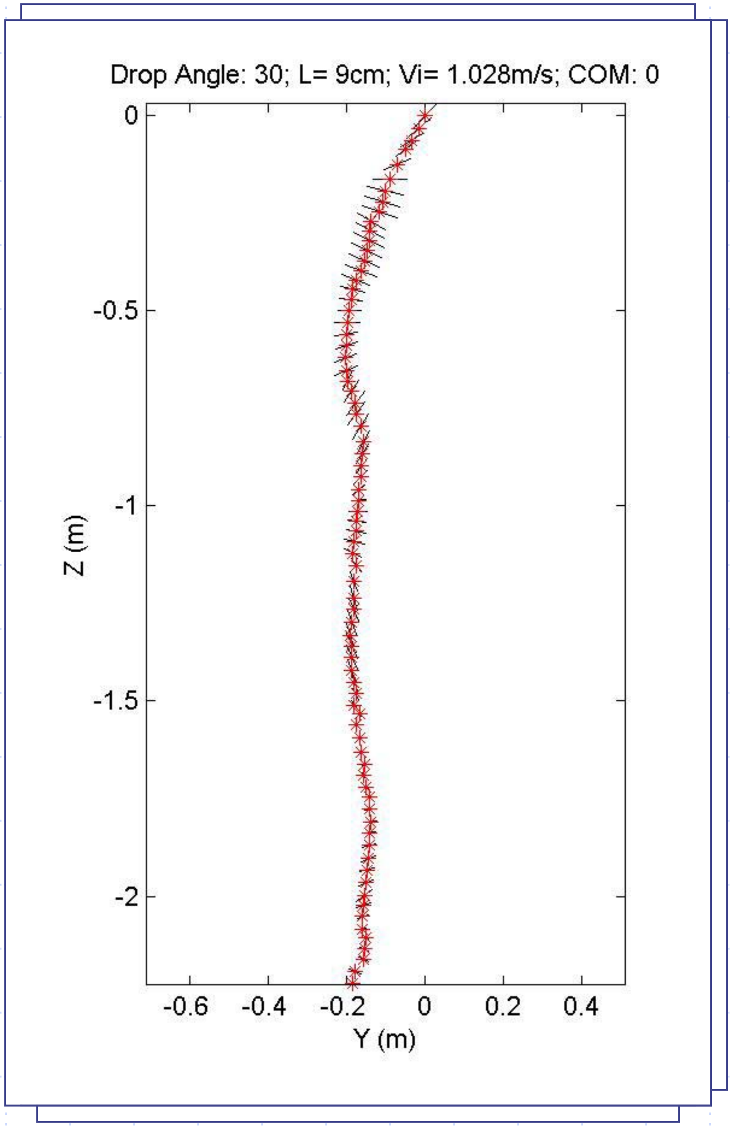


Impact Angle Frequency of Occurrence



Trajectory Patterns

1. Straight
2. Slant
3. Spiral
4. Flip
5. Flat
6. See Saw
7. Combination



Multiple Linear Regression

- General Multiple Linear Regression Equation:

$$f_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \varepsilon_i$$

- Used least squares solution to determine correlation coefficients.
- Input: cos(drop angle); L/D; V_{ind}; COM_{nd}
- Output: (x_m, y_m, z_m, Psi, u, v, w)

Multiple Regression Results

	x_m	y_m	Psi	u	v	w
β_0	-.0746	-.0546	102.5691	.0040	-.0135	-.9481
β_1	.1190	-.0828	-13.3508	-.0075	-.0106	-.1080
β_2	-.0469	-.0798	-.5009	-.0011	.0005	.0295
β_3	.0372	.0622	1.0437	.0025	.0011	-.0221
β_4	.2369	.4330	472.2135	-.0090	.0537	-1.2467

- Most important parameter for impact prediction is Psi (impact angle).

Check of regression equation:

Determine Psi for case where:

L=15cm, V_i = 3m/s, COM = 2, Drop Angle = 15°

Yields: Psi = 181.2°

For COM = 1: Psi = 136.1°

For COM = 0: Psi = 90.4°

Results from MIDEX

- COM position is the most influential parameter for predicting a mine's impact position and angle.
- Final velocities were lowest for COM 0 cases due to the increased effect of hydrodynamic drag.
- Trajectories became more complex as L/D decreased (9 cm mine rotated about z-axis).
- Observed trajectory patterns were more complex than those assumed by IMPACT 25/28. Accurate representation of a mine's water phase motion requires both momentum and moment of momentum equations.

Future Plan: Ensemble IMPACT Model

- Update IMPACT25/28 using statistical-dynamical approach (Stochastic-Dynamic Model)

- Dynamical component: to improve IMPACT25/28 with correct physics (Hydrodynamics and sediment dynamics)

- Statistical component: to incorporate unknown environmental parameters (ensemble modeling)

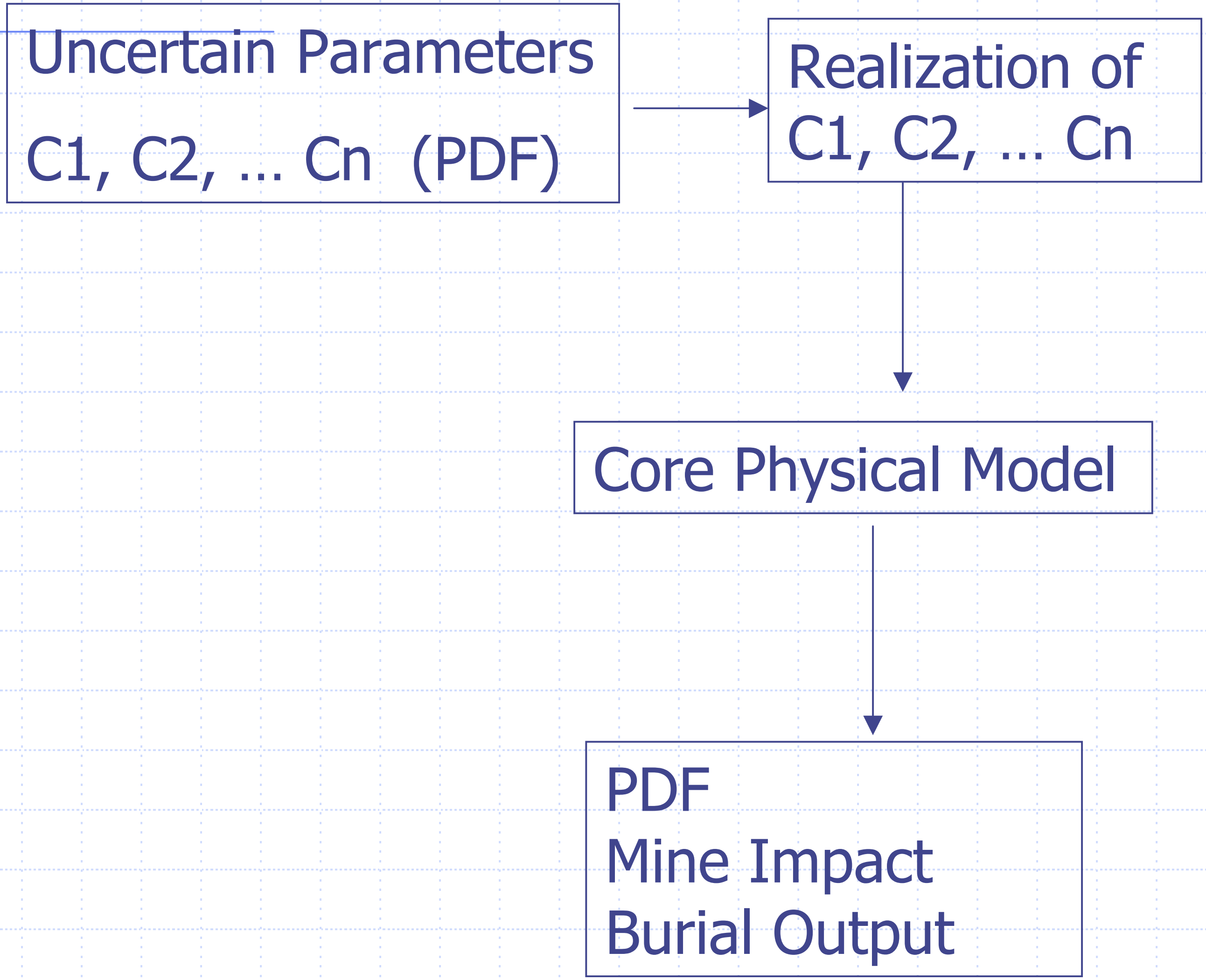
Two Types of Model Parameters

- Model parameters: L/D, $_{m}$, COM, CB
- Mine shape (small scale experiments are useful)
- Stochastic parameters: drop velocity V_{in}, drop height angle (uncertainty)
- Stochastic parameters: environmental variables such as waves, currents, etc. (uncertainty)
- Stochastic parameters: sediment density and shear strength profiles (uncertainty)

Ensemble Approach

- Uncertain parameters
- MIW perspective: even the most accurate physics model, by itself, is of minimal value to the operational community (many uncertain parameters)

Ensemble IMPACT Model



Benefits of Developing Ensemble IMPACT Model

- Building block of Expert System
- Incorporating Navy data into the model
- Model output useful to the Navy

Conversion of Navy Data into Density and Shear Strength

- MIW sediment data: grain size (240 categories), multi-modes (ex: silt and rock)
- Penetrometer measurements
- The relationship between Navy's data and model input (density and shear strength) is statistical.